

**B.Sc. (Honours) Examination, 2019**  
**Semester-II**  
**Statistics**  
**Course : BSC-22**  
**(Real Analysis and Numerical Analysis)**

**Time : 3 Hours**

**Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) State and prove Cantor's theorem on nested intervals. 5  
 b) Prove that the sequence defined by  $x_{n+1} = x_n(2 - x_n), 0 < x_1 < 1, x_n > 0, \forall n$  is convergent. 5
2. a) If  $\{x_n\} \rightarrow l, \{y_n\} \rightarrow m$ , then show that  $\{x_n^2 + y_n\} \rightarrow l^2 + m$ . 5  
 b) Let  $\{u_n\}$  and  $\{v_n\}$  be bounded sequence. Then show that  $\overline{\lim} u_n + \overline{\lim} v_n \geq \overline{\lim}(u_n + v_n)$ . 5
3. Discuss the convergence of the following series  
 a)  $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$     b)  $\left(\frac{1}{2}\right)^2 + \left(\frac{1.3}{2.4}\right)^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 + \dots$  5+5
4. a) Establish the convergence and find the limit of the sequence  $u_n = \left(1 + \frac{1}{n}\right)^n$ . 5  
 b) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ . 5
5. a) Derive the relation between  $\Delta$  & E operator. Also find  $\frac{\Delta}{E} \sin x$ . 5  
 b) Find  $n^{\text{th}}$  order finite difference of  $e^{bx} + ax^2$ . 5
6. a) Describe Newton's backward interpolation formula. 5  
 b) Using the method of separation of symbols (on otherwise) show that  $u_1x + u_2x^2 + u_3x^3 + \dots = \frac{x}{1-x}u_1 + \frac{x^2}{(1-x)^2}\Delta u_1 + \frac{x^3}{(1-x)^3}\Delta^2 u_1 + \dots$ , where,  $u_n = Eu_{n-1}$ . 5
7. a) Describe the method of iteration and its convergence criterion. 5  
 b) Describe Simpson's 1/3 rd rule for numerical integration. 5

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