M.A./M.Sc. Examination 2018

Semester - I
Mathematics
Course: MMC-16 (Old)
(Methods of Applied Mathematics)
(For Back Candidates)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer question no. 1 and ***three*** from the rest.

1. Answer *any five* questions: 5×2=10

   i) Find the Laplace transform of $f(t)$,

   where $f(t)=\begin{cases} 3t & 0<t<2 \\ 6 & 2<t<4 \end{cases}$

   and $f(t+4)=f(f)$.

   ii) Find the inverse Laplace transform of $F(P)$,

   where $F(P)=\dfrac{P^2-7P+11}{(P-1)(P-2)(P-4)}$.

   iii) Find the value of λ for which the integral equation

   $y(x)=e^x+\lambda\int_0^1 5e^x e^t y(t)\,dt$ has infinitely many solutions.

   iv) Find the value of $u(5)+u(7)$, where $u(x)$ is the solution of the Volterra integral equation $1+x-e^x=\int_0^x (t-x)u(t)\,dt$.

   v) Solve the Fredholm integro-differential equation

   $\dfrac{du}{dx}=\sec^2 x-\ell n(2)+\int_0^{\pi/3} u(t)\,dt,\ u(0)=0.$

   vi) Starting with initial approximation $y_0(x)=x^2$, find the third approximation for the integral equation $y(x)=\dfrac{x^3}{2}-2x-\int_0^x y(t)\,dt$.

   vii) Find the finite sine transform of $\sin ax$.

   viii) If $L\{f(t)\}=F(s)$,

   then prove that $L\left\{\dfrac{f(t)}{t}\right\}=\int_s^\infty F(x)\,dx$.

2. i) With the help of Hilbert-Schmidt theorem, solve the integral equation

   $y(x)=1+\dfrac{2}{\pi}\int_0^\pi \cos(x+t)\,y(t)\,dt.$ 5

P.T.O.

ii) Find the resolvent kernel of the Volterra integral equation

$$y(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t} y(t)\,dt.$$

Hence find the value of $y(\pi/2)$. 5

3. i) Use Laplace transforms to solve the initial valve problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y = h(t), y(0)=0, y'(0)=1$$ 5

where $h(t) = \begin{cases} 1, & 0 < t < \pi/2 \\ 0, & t > \pi/2 \end{cases}$.

ii) Apply convolution theorem to prove that $\beta(m,n) = \dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$, $m,n > 0$. 3

iii) Evaluate $\displaystyle\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt$. 2

4. i) State Fredholm alternative theorem. Using the theorem find the conditions for which the integral equation

$$y(x) = f(x) + \lambda\int_0^1 (1-3xt)\,y(t)\,dt$$ has infinitely many solution. 1+4

ii) Solve the integral equation

$$y(x) = 1 - x\cos x + x + x^2 + \sin x - \int_0^x y(t)\,dt$$ by using modified Adomian decomposition method. 2

iii) Solve the first kind Volterra integral equation $\displaystyle\int_0^x 3^{x-t} y(t)\,dt = x$. 2

iv) Give an example of a function which is not of exponential order but whose Laplace transform exits. 1

5. i) Find $f(x)$, where $\displaystyle\int_0^\infty f(x)\cos(\lambda x)\,dx = \begin{cases} 1-\lambda, & 0 \le \lambda < 1, \\ 0, & \lambda > 1 \end{cases}$. 2

ii) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| < 1, \\ 0, & |x| > 1 \end{cases}$. 5

Hence evaluate $\displaystyle\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos(x/2)\,dx$.

iii) Using Parseval's identity, Prove that $\displaystyle\int_0^\infty \frac{t^2}{(t^2+1)^2}\,dt = \pi/4$. 2

iv) If $f(t) = \begin{cases} e^{-xt}\varphi(t), & t > 0 \\ 0, & t < 0 \end{cases}$, then

Prove that $F\{f(t)\} = L\{\varphi(t)\}$. 1

6. i) Find the necessary condition for the existence of the extremum of the functional

$$I[y(x)] = \int_{x_0}^{x_1} f(x, y, y', y'')\,dx.$$ 4

ii) Find the shortest distance between the parabola $y = x^2$ and the straight line $y = x - 5$. 4

iii) Find the extremum of the functional

$$I[y(x)] = \int_0^e (xy'^2 + yy')\,dx.$$

satisfying $y(1) = 0$ and $y(e) = 1$. 1