M.A./M.Sc. Examination 2018

Semester - I

Mathematics

Course: MMC-16 (New)

(Integral Transforms and Integral Equations)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer question no. 1 and ***three*** from the rest.

1. Answer *any five* questions: 5×2=10

 i) Find the Laplace transform of $f(t)$,

 where $f(t)=\begin{cases} 3t & 0<t<2 \\ 6 & 2<t<4 \end{cases}$

 and $f(t+4)=f(f)$.

 ii) Find the inverse Laplace transform of $F(P)$,

 where $F(P)=\dfrac{P^2-7P+11}{(P-1)(P-2)(P-4)}$.

 iii) Find the value of λ for which the integral equation

 $y(x)=e^x+\lambda\int_0^1 5e^x e^t y(t)\,dt$ has infinitely many solutions.

 iv) Find the value of $u(5)+u(7)$, where $u(x)$ is the solution of the Volterra integral equation $1+x-e^x=\int_0^x (t-x)u(t)\,dt$.

 v) Solve the Fredholm integro-differential equation

 $\dfrac{du}{dx}=\sec^2 x-\ell n(2)+\int_0^{\pi/3} u(t)\,dt,\ u(0)=0.$

 vi) Starting with initial approximation $y_0(x)=x^2$, find the third approximation for the integral equation $y(x)=\dfrac{x^3}{2}-2x-\int_0^x y(t)\,dt$.

 vii) Find the finite sine transform of $\sin ax$.

 viii) If $L\{f(t)\}=F(s)$,

 then prove that $L\left\{\dfrac{f(t)}{t}\right\}=\int_s^\infty F(x)\,dx$.

2. i) With the help of Hilbert-Schmidt theorem, solve the integral equation

 $y(x)=1+\dfrac{2}{\pi}\int_0^\pi \cos(x+t)\,y(t)\,dt.$ 5

P.T.O.

ii) Find the resolvent kernel of the Volterra integral equation

$$y(x)=e^{x}\sin x+\int_0^x \frac{2+\cos x}{2+\cos t}\, y(t)\,dt.$$

Hence find the value of $y(\pi/2)$. 5

3. i) Use Laplace transforms to solve the initial valve problem

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+5y=h(t),\ y(0)=0,\ y'(0)=1,$$ 5

where $h(t)=\begin{cases}1, & 0<t<\pi/2\\ 0, & t>\pi/2\end{cases}$.

ii) Apply convolution theorem to prove that $\beta(m,n)=\dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$, $m,n>0$. 3

iii) Evaluate $\displaystyle\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt$. 2

4. i) State Fredholm alternative theorem. Using the theorem find the conditions for which the integral equation

$y(x)=f(x)+\lambda\displaystyle\int_{-1}^{1}\cos\pi(x-t)\,y(t)\,dt$ has infinitely many solution. 1+4

ii) Solve the integral equation

$y(x)=1-x\cos x+x+x^2+\sin x-\displaystyle\int_0^x y(t)\,dt$ by using modified Adomian decomposition method. 2

iii) Solve the first kind Volterra integral equation $\displaystyle\int_0^x 3^{x-t}y(t)\,dt=x$. 2

iv) Give an example of a function which is not of exponential order but whose Laplace transform exits. 1

5. i) Find $f(x)$, where $\displaystyle\int_0^\infty f(x)\cos(\lambda x)\,dx=\begin{cases}1-\lambda, & 0\le\lambda<1,\\ 0, & \lambda>1\end{cases}$. 2

ii) Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|<1,\\ 0, & |x|>1\end{cases}$. 5

Hence evaluate $\displaystyle\int_0^\infty\left(\frac{x\cos x-\sin x}{x^3}\right).\cos(x/2)\,dx$.

iii) Using Parseval's identity, Prove that $\displaystyle\int_0^\infty\frac{t^2}{(t^2+1)^2}\,dt=\pi/4$. 2

iv) If $f(t)=\begin{cases}e^{-xt}\varphi(t), & t>0\\ 0, & t<0\end{cases}$, then

Prove that $F\{f(t)\}=L\{\varphi(t)\}$. 1

6. i) If $f(k)$ is the Fourier transform of $F(x)$, then prove that

$$\int_{-\infty}^{\infty}|f(k)|^2\,dx=\int_{-\infty}^{\infty}|F(x)|^2\,dx$$ 2

ii) If $L\{f(t)\}=F(s)$ and $g(t)=\begin{cases}0, & t<a\\ f(t-a), & t>a\end{cases}$

then prove that $L\{g(t)\}=e^{-as}F(s)$. 2

iii) Prove that the resolvent kernel $R(x,t:\lambda)$ of the integral equation

$y(x)=f(x)+\lambda\int_a^b k(x,t)\,y(t)\,dt$, satisfies the integral equation

$$R(x,t;\lambda)=f(x)+\lambda\int_a^b k(x,z)\,R(z,t;\lambda)\,dz.$$ 2

iv) Convert the integral equation

$y(x)+\dfrac{\lambda}{2}\displaystyle\int_0^1|x-s|\,y(s)\,ds=ax+b$ into a differential equation. 3

v) Find $y(0)$, where $y(x)=1+\int_0^x y(t)\,dt$. 1

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