M.A./M.Sc. Examination 2018

Semester - I

Mathematics

Course: MMC-15 (Old)

(Classical Mechanics)

(For Back Candidates)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four***.

1. Derive relations among velocities and accelerations of the motion of a particle observed with reference to two frames, one is rotating relative to other having common origin. 5

2. Obtain the equation of motion of a particle moving under the action of gravity near the Earth surface (take the rotation of the Earth about its axis into account). 5

3. Derive expressions of kinetic energy and angular momentum involved in the rotational motion of a rigid body. 3+2

4. Explain Euler's angle. Hence derive expression for angular velocity of rotational motion of rigid body in terms of Euler's angle and their time derivative. 3+2

5. Derive Euler's equation of motion of rigid body and find its solution when the body is symmetric. 5

6. Define generalized coordinate of mechanical system? State D'Alembert's principle. Derive Lagrange's equation of motion of second kind. 1+1+3

7. State Legendre's theorem in its general form and use result of this theorem of derive Hamilton's equation of motion of a holonomic mechanical system. 2+3

8. Define generalized coordinate and obtain kinetic energy, potential energy, Lagrangian, Legrange's equation of motion and its solution for the motion of a particle along a line under the influence of a force directed towards a fixed point (center) on the line with a magnitude proportional to the distance of the particle from the center. 5

9. Show that Lagrangian of a system with holonomic constraints is not unique. Hence verify whether $L(x,\dot{x})=\frac{1}{2}\dot{x}^2$ and $L(x,\dot{x})=\log(\dot{x})$ may be treated a inequivalent Lagrangian for the free motion of a particle along a line. 2+1+2

10. Define Hamilton's principal function. Derive equation for it. Show that Hamilton's principal function may be regarded as generating function of a special type of canonical transformations. Discuss the significance of the transformation. 1+1+2+1

P.T.O.

11. Define Poisson bracket. Show that Poisson bracket follows Jacobi identity. 1+4

12. What do you mean by a constant of the motion of a mechanical system? Prove that Poisson bracket of two constant of motion is also a constant of the motion. 1+4

13. Study the small oscillation around the stable equilibrium point of a mechanical system moving under the influence of a force described by the potential

$$V(x, y) = \frac{1}{2}(x^2 + y^2) - xy.$$

1+4

14. Write down the correspondence among generalized coordinate, momenta and associated operators on Hilbert space for the quantum mechanical description of motion of a particle on a line. Hence derive the Schrödinger equation for this system.

2+3