M.A./M.Sc. Examination 2018
Semester - I
Mathematics
Course: MMC-15 (New)
(Partial Differential Equations)

Time: Three Hours　　　　Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) Is the solution of the Cauchy problem $p+q=u^2$ with $u(x,0)=1, -\infty<x<\infty$ unique? Justify your answer. $\left[u=u(x,y),\ p=\frac{\partial u}{\partial x},\ q=\frac{\partial u}{\partial y}\right]$ 2+1=3

   b) Find a solution of $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2},\ -\infty<x<\infty, t>0,$ that satisfies the given initial condition: $u(x,0)=\sin x,\ \left.\frac{\partial u}{\partial t}\right]_{(x,0)}=a\cos x,\ -\infty<x<\infty.$ 2

   c) Reduce the PDE $\frac{\partial^2 u}{\partial x^2}+x\frac{\partial^2 u}{\partial y^2}=0,\ x<0\ \forall x,y$ to canonical form. 5

2. a) Find the solution of the PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=e^{-x}\cos y,$ which tends to zero as $x\to\infty$ and has the value $\cos y$ when $x=0.$ 5

   b) Solve the following PDE by using Monge's method:
   $$\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}=\left(\frac{\partial u}{\partial y}\right)^3.$$ 5

3. a) Use a separable solution $u(x,y)=f(x)+g(y)$ to solve the PDE
   $$\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial u}{\partial y}+x^2=0.$$ 2

   b) What do you mean by Neumann type boundary conditions for a PDE? Hence prove that if the Neumann problem for a bounded region has a solution, then it is either unique or it differs from one another by a constant only. 1+2=3

   c) Use the method of characteristics to find the solution of the first order PDE
   $$x^2\frac{\partial u}{\partial x}+xy\frac{\partial u}{\partial y}=u^2$$ 5
   which passes through the curve $u=1,\ x=y^2$. Determine where this solution becomes singular.

P.T.O.

4. a) Using the Laplace transform method, solve the following IBVP: 5

$$\text{PDE}: \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 2018,\ t > 0,$$

$$\text{BC's}: u(0,t) = u(2018,t) = 0,\ t > 0,$$

$$\text{IC's}: u(x,0) = \sin(\pi x), \left.\frac{\partial u}{\partial t}\right]_{(x,0)} = -\sin(\pi x),\ 0 < x < 2018.$$

b) Use finite Fourier sine transform to solve: 5

$$\nabla^2 u = 0,\ \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

with

$$u(x,0) = 0,\ u(x,\pi) = 2018,$$

$$u(0,y) = 0 = u(\pi,y) \text{ for every } y,$$

$$u(x,y) \text{ is bounded.}$$

5. a) State and prove maximum-minimum principle for a function $u(x,y)$ which is continuous in a closed region $\overline{\mathbb{R}}$ and satisfies the Laplace equation $\nabla^2 u = 0$ in the interior of $\mathbb{R}$. 4

b) Obtain the solution by separation of variables method of the following IBVP for the PDE: 6

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial u}{\partial t},\ 0 < x < L,\ t > 0,$$

where

$$u(x,0) = 2\sin\left(\frac{3\pi x}{L}\right),\ 0 \le x \le L,$$

$$u(0,t) = 0 = u(L,t) \text{ for } t > 0.$$

6. a) Write the Hadamard's conditions for a well-posed PDE. 2

b) Solve the following exterior Dirichlet problem for a circle: 8

$$\text{PDE}: \nabla^2 u = 0,\ r > a,\ 0 \le \theta < 2\pi,$$

$$\text{BC}: u(a,\theta) = f(\theta),\ 0 \le \theta < 2\pi,$$

u must be bounded as $r \to \infty$.