

M.A./M.Sc. Examination 2018

Semester – I

Mathematics

Course: MMC-14 (Old)

(Ordinary Differential Equations)

(For Back Candidates)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings.

Answer *any four* questions.

1. a) Write four different solutions of the initial value problem

$$\frac{dy}{dx} = xy^{\frac{1}{2}}, \quad y(0) = 0. \quad 2$$

- b) Consider the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

State the existence and uniqueness theorem for the above problem. 2

- c) Consider the problem

$$\frac{dy}{dx} = y^2, \quad y(1) = -1.$$

Find the “best possible” interval according to existence and uniqueness theorem, in which the initial value problem possesses a unique solution. 4

- d) Give an example of a function $f(x, y)$, so that $f(x, y)$ does not satisfy Lipschitz condition but the solution of the differential equation is unique in a domain D . 1

- e) Give an example of a function $f(x, y)$, so that $f(x, y)$ satisfies Lipschitz condition but $\frac{\partial f}{\partial y}$ does not exist in a domain D . 1

2. a) Using Green’s function solve the initial value problem

$$y'' - y = \frac{1}{x}, \quad y(1) = 0, \quad y'(1) = 0. \quad 3$$

- b) Find the Green’s function for the boundary value problem

$$y'' + 4y = 3, \quad y'(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 0$$

and hence solve the problem. 4

- c) Show that the boundary value problem

$$x'' + 16x = 0, \quad x(0) = 0, \quad x(\alpha) = 0$$

can have unique, many or no solutions depending on the value of α . 3

3. a) Find the eigen values and eigen functions of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(1) + y'(1) = 0. \quad 5$$

- b) Find the general solution of the nonhomogeneous linear system

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3t \\ e^{-t} \end{pmatrix} \text{ on } (-\infty, \infty). \quad 5$$

P.T.O.

4. a) Let $\{f, g\}$ and $\{F, G\}$ be two fundamental sets of the differential equation

$$a_0(t)x'' + a_1(t)x' + a_2(t)x = 0 \text{ on } a \leq t \leq b. \quad 3$$
 Prove that $W(f, g) = CW(F, G)$, $C \neq 0$ is a constant.
- b) Let f and g be real linearly independent solutions of

$$\frac{d}{dt} \left[P(t) \frac{dx}{dt} \right] + Q(t)x = 0 \text{ on the interval } a \leq t \leq b.$$
 Prove that between any two consecutive zeros of f there is precisely one zero of g . 4
- c) Transform the equation

$$\frac{d^2x}{dt^2} - \tan t \frac{dx}{dt} + x = 0$$
 into an equivalent self-adjoint equation. 3
5. a) Find the third approximation of the solution of the equation

$$\frac{dy}{dx} = 2 - \frac{y}{x}$$
 by Picard's method with initial condition $y(1) = 2$. 2
- b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. 2
- c) Prove that $\frac{d}{dx} \{x^n J_n(x)\} = x^n J_{n-1}(x)$. 4
 and $\frac{d}{dx} \{x^{-n} J_n(x)\} = -x^{-n} J_{n+1}(x)$.
- d) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. 2
6. a) Solve the Legendre's equation

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$
 about the point $x = 0$. 4
- b) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.
 Using the above formula find the values of $P_3(3)$. 4
- c) Compute e^{At} , where $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. 2
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