M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (P11/A13) (New)

(Theory of Computation-I)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt ***any four*** questions.

1. a) Define Prenex Normal Form of a formula. Obtain the same for the form
$$(\forall x)(\forall y)\left[(\exists z)\left(P(x,z)\wedge P(y,z)\right)\to(\exists u)Q(x,y,u)\right].$$ 1+1=2

   b) Define Skolem standard form of a formula. Let *S* be the set of clauses representing the standard form of a formula *F*. Show that *F* is inconsistent if and only if *S* is inconsistent. 1+3=4

   c) Define Herbrand universe, Herbrand base and interpretation over Herbrand universe. For the formula
$$S=\left\{P(x)\vee Q(x),R\left(f(y)\right)\right\}$$ 1+1+1+1=4
write a *H*-interpretation *I*.

2. a) State and prove Davis and Putnam's Pure-literal rule. 1+2=3

   b) Show that a set *S* of clauses is unsatisfiable if and only if there is a finite unsatisfiable set S' of ground instances of clauses of *S*. 5

   c) For the set $S=\left\{P(x),\ Q\left(x,f(x)\right)\vee\sim P(x),\ \sim Q\left(g(y),z\right)\right\}$
find an unsatisfiable set S' of ground instances of clauses in *S*. 2

3. a) State the Resolution principle. Show, by resolution, that a set *S* of clauses is unsatisfiable if and only if there is a resolution – deduction of the empty clause from *S*. 1+4=5

   b) Use resolution to show the validity of the following argument: 5
Custom officials searched everyone who entered this country who was not a V.I.P. Some of the drug-pushers entered this country and they were only searched by drug-pushers. No drug-pusher was a V.I.P. Therefore, some officials were drug-pushers.

4. a) Define a sequential machine *S* without output. Show that for a *n*-state machine (finite) *S*
$$(\forall s)_{S^c}\ (\exists x)_{\Sigma_k^*}\ \left(s=rp_S(x)\wedge lg(x)<n\right)$$ 1+3=4
where S^c is the connected submachine of *S*.

   b) Let *S* be a finite state sequential machine without output and *R* be a congruence relation on the set *S*. Define the quotient machine S/R and show that it is well defined. 1+1=2

P.T.O.

c) Let ϕ be a homomorphism from a machine S into a machine T. Show that it is possible to define a congruence relation R on S so that T is isomorphic to S/R. 4

5. a) Define $T(R,\beta)$, the quotient sequential machine with output modulo-R and parameter β. R_1, R_2 are two right congruence relations on $\sum_k^*$ which respectively refines β_1 and $\beta_2 \subseteq \sum_k^*$. Show that there exists a homomorphism from

$$T(R_1,\beta_1) \text{ to } T(R_2,\beta_2)$$

if and only if

$$R_1 \subseteq R_2 \text{ and } \beta_1 \subseteq \beta_2 .$$ 1+4=5

b) Define equi-response relation $\perp(S)$ of a sequential machine S without output. Show that it is a right congruence relation on S. Show also that for any connected machine S,

$$T(\perp(S)) \text{ is isomorphic to } S.$$ 1+1+3=5

6. a) Define R_F, the equivalence relation for states of a single machine $S = \langle S, \sum_k, M, a, F \rangle$. Show that R_F is a right congruence relation on S which refines F. Show also that R_F is the largest congruence relation which refines F.

1+1+3=5

b) Let S and T be two connected machines. $S \equiv T$ if and only if

S^M and T^M are isomorphic. 5

———