M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (P5) (New)

(Algebraic Coding Theory-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) Define a binary symmetric channel (BSC). Suppose that codewords from the code $\{000, 111\}$ are being sent over a BSC with crossover probability $p = 0.05$. Suppose that the word 110 is received. Find P(110 received/111 sent). 1+2

   b) Show that a code C is n-error-correcting if and only if $d(C) \geq 2n+1$. 3+4

2. a) Let C be a linear code over F_q. Show that $d(C) = W(C)$. 4

   b) Let C be an $[n, K]$-linear code over F_q, with generator matrix G. For $v \in F_q^n$, $vG^T = O$ if and only if $v \in C^+$. Moreover an $(n-K) \times n$ matrix H is a parity check matrix for C if and only if the rows of H are linearly independent. 3+3

3. a) Let C be an $[n, K, d]$- linear code over a finite field F_q, and H be a parity check matrix for C. For $u, v \in F_q^n$, show that u and v are in same coset if and only if u, v have same syndrome. 6

   b) Show that there is no self dual binary code of parameters [10, 5, 4]. 4

4. a) Show that a binary (n, M, d) – code exists if and only if a binary $(n+1, M, d+1)$ - code exists, where d is odd. 6

   b) Find the distance of $Ham(\gamma, 2)$. Hence or otherwise show that $Ham(\gamma, 2)$- is exactly single error-correcting. 3+1

5. Define extended binary Golay code G_{24}. Give an application of extended Golay code. Show that G_{24} is an exactly three-error-correcting code. 2+1+7

6. a) Construct a binary [9, 3, 4] – linear code. 5

   b) Consider the code $C = \{\lambda(1, \alpha, \alpha+1) : \lambda \in F_q\}$ over F_q, where α is a root of $2 + x + x^2 \in F_3[x]$. Find $Tr_{F_9/F_3}(C)$. 5