M.A./M.Sc. Examination 2018
Semester - III
Mathematics
Optional Course: MMO-31 (P3) (New)
(Advance Real Analysis-I)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer Question No. 6 and ***any three*** from the rest.

1. a) Define $C^{+}(f, a)$, the cluster set of f at a from the right. Show that $C^{+}(f, a)$ is closed. 1+3

   b) For any function f show that the following relations hold except on a countable set
   $$\overline{f}(x+0) = \overline{f}(x-0) \geq f(x) \geq \underline{f}(x+0) = \underline{f}(x-0).$$ 4

   c) Show that the set of all points of discontinuities of any function is an F_σ-set. 4

2. a) State and prove Zygmond theorem. 4

   b) If one of the Dini derivates of a continuous function f is continuous at x_0 then show that $f'(x_0)$ exists. 4

   c) Define cantor function on [0, 1] and show that it is continuous on [0, 1]. 4

3. a) Let f be a Lebesgue integrable function on [a, b] and let
   $$F(x) = \int_a^x f \text{ for } x \in [a, b],$$
   Show that $F' = f$ a.e on $[a, b]$. 6

   b) If f is absolutely continuous on [a, b] then show that f satisfies the Luzin condition in [a, b]. 4

   c) If f is absolutely continuous on $\mathbb{R}$ and $D^{+}f \geq 0$ a.e then f is non-decreasing. 2

4. a) Let $f : [a, b] \to \overline{\mathbb{R}}$, show that f is u.s.c. at $\xi \in [a, b]$ if and only if
   $$f(\xi) \geq \overline{f}(\xi) = \max\left\{\overline{f}(\xi+0), \overline{f}(\xi-0)\right\}.$$ 5

   b) Let $\{f_n\}$ be a sequence of Baire I functions converges uniformly to f on $\mathbb{R}$ show that f is also a Baire I function. 5

   c) Give an example with justifications of a Baire 2 function which is not Baire-I. 2

5. a) If f is a Baire Class I function then for each α show that $\{x : f(x) < \alpha\}$ and $\{x : f(x) > \alpha\}$ are of F_σ-sets. 5

   b) If f is a convex function on [a, b] then prove that f is bounded on [a, b] and continuous on (a, b). 2+2

   c) If f has finite number of points of discontinuities in [a, b], then show that f is Baire-I on [a, b]. 3

6. Answer ***any two*** of the following: 2x2

   a) Show that $f(x) = \sin x + \cos x$ is absolutely continuous on [0, 1].

   b) Prove or disprove
   The sum of two convex functions both defined on an interval I is convex on I.

   c) Define Vitali cover. State Vitali covering theorem.