

M.A./M.Sc.Examination, 2018
Semester - III
Mathematics
Course: MMO-31 (A10) (New)
(Nonlinear Differential Equation-I)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meaning.
Answer *any four* questions.

1. What do you mean by the conservation law of KdV equation? Derive the first three conservation laws of KdV equation and modified KdV equation. 2+(4+4)=10

2. a) Define ColHopf transformation. Use ColHopf transformation to solve the Burger's equation:
$$u_t + uu_x - \varepsilon u_{xx} = 0 \quad 0 \leq x < \pi$$
$$u(x, 0) = f(x)$$
$$u(0, t) = u(\pi, t) = 0 \quad t > 0.$$

b) Explain Muare transformation. 6+4=10

3. Explain the difference between regular perturbation method and singular perturbation method. Solve the pendulum problem using two timing scale. 4+6=10

4. Use perturbation method, solve the initial value problem
$$\frac{d^2x}{dt^2} = -\frac{1}{(1 + \varepsilon x)^2} \quad 0 < t$$
$$x(0) = 0;$$
10
$$\left. \frac{dx}{dt} \right|_{x=0} = 1$$

5. Write down Van der Pol equation with very small nonlinearity. For a small nonlinearity explain the existence of periodic motion and stability of the system. Hence obtain the amplitude of the limit cycle. 1+5+4=10

6. Starting from fluid equations, using reductive perturbation technique obtain the KdV equation of a plasma model of your choice. 10