

M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (A8) (New) (Magnetohydrodynamics-I)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt *any four* questions.

1. a) Discuss the phenomenon of Debye shielding in a plasma. 4
b) Calculate the potential of charged particle in a plasma at a distance x from the origin at which the potential is ϕ_0 . When a plasma is said to be quasineutral? 3+1
c) Write down the Maxwell's equations in electromagnetic theory. 2
2. a) Discuss the motion of a charged particle in a uniform magnetic field. Hence show that the particle's trajectory is a helix. 4
b) Prove that for the motion of a charged particle under the uniform electromagnetic fields, the total energy is conserved. Discuss the case of time varying electromagnetic fields. 4+2
3. a) Define magnetic moment of a charged particle. Show that the magnetic flux of a charged particle is an adiabatic invariant. 2+4
b) Obtain the dispersion relation for Langmuir waves in a plasma and discuss their characteristics. 3+1
4. a) Show that for the propagation of electromagnetic waves perpendicular to the external magnetic field, the dispersion relation is given by

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2 (\omega_p^2 - \omega^2)}{\omega^2 (\omega_h^2 - \omega^2)}, \quad 6$$

where ω_p and ω_h are the plasma frequency and upper hybrid wave frequency to be defined by you, and other symbols have their usual meanings.

Hence obtain the phase velocity and group velocity of the wave. What are the cut-off and resonance frequencies of this wave?

5. Consider the propagation of electromagnetic waves in an unmagnetized electron-ion plasma in which ions are immobile and electrons have no thermal motion. Show that the wave frequency is given by $\omega^2 = \omega_p^2 + c^2 k^2$. Compare this relation with that for Langmuir waves and discuss. 6+4
6. For the propagation of large amplitude ion-acoustic waves in an unmagnetized plasma with cold ions and Boltzmann distributed electrons obtain the energy-like equation for a pseudo particle of unit mass. Hence discuss the conditions for the existence of solitary waves. 6+4