M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (A7/P8) (New)

(Lie Theory of Ordinary and Partial Differential Equations)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer **Question No.1** and ***any three*** from rest five.

1. Define Lie group and Lie group of point transformations. Point out the important difference between these two. Show that the set of matrices $\left\{\begin{pmatrix} 1 & a & c \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}; a,b,c \in \mathbb{R}\right\}$ is a Lie group. 2+2+2+7

Or

Show that $y''(x)+p(x)y'(x)+q(x)y(x)=h(x); p(x),h(x)\in C(\mathbb{R})$ admits Lie group of point transformations generated by $\hat{X}_1=\phi_1(x)\frac{\partial}{\partial y}$, $\hat{X}_2=\phi_2(x)\frac{\partial}{\partial y}$ where two functions $\phi_1(x)$ and $\phi_2(x)$ are solutions of a differential equation to be determined by you. Use this information to recover the solution (obtained by the method of variation of parameters) of the given equation. 5+8

2. What do you mean by Lie group of point transformations in infinitestimal form and define infinitestimal generator for one-parameter Lie group of point transformations. Find infinitesimal generators for the Lie group of point transformations $(x,y)\to(x^*,y^*)=\left(\frac{\varepsilon_3 x+\varepsilon_4 y+\varepsilon_5}{\varepsilon_1 x+\varepsilon_2 y}, \frac{\varepsilon_6 x+\varepsilon_7 y+\varepsilon_8}{1+\varepsilon_1 x+\varepsilon_2 y}\right)$. Is it possible to get the grop of rotations in the (x,y)-plane from the given group. Justify your answer. 1+1+5+2

3. Discus invariant curve (surface), invariant family of curves (surfaces) and canonical variab;es involved with the Lie group of point transformations. State the principle for their determination. Find canonical variables for the Lie group of point transformations $(x,y)\to(x^*,y^*)=(x\cosh\varepsilon+h\sinh\varepsilon,\ x\sinh\varepsilon+y\cosh\varepsilon);\ \varepsilon\in\mathbb{R}$. 1+1+1+2+4

4. State contact conditions relating the differentials $dx,dy,dy_1,\ldots,dy_k$ in the geometric space comprising independent and dependent variables x and y respectively. Hence derive formulas for $\eta^{(i)}, i\in\mathbb{N}$ involved in the prolongation of infinitesimal generator $\hat{X}=\xi(x,y)\frac{\partial}{\partial x}+\eta(x,y)\frac{\partial}{\partial y}$ for a Lie group of point transformations. 2+7

5. Verify whether $y''(x) = \frac{1}{y^3}$ admits Lie group of point transformations generated by $\hat{X}_1 = \frac{\partial}{\partial x}$, $\hat{X}_2 = 2x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$. Verify whether the Lie algebra ε is solvable and solve this equation, if yes. 2+2+1+4

6. Find infinitesimal generators for the Lie group of (point) symmetry transformations admitted by the Burger's $\varepsilon q, u_t + uu_x = u_{xx}$. 9

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