M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (A5) (New)

(Dynamics of Ecological System-I)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt ***any four*** questions.

1. a) Sketch the phase diagram and obtain the dynamic solutions of the following autonomous system: 4

$$\frac{du}{dt} = 4u - 5v,$$

$$\frac{dv}{dt} = 5u - 4v.$$

b) In *2D* autonomous system, it is well known that there can be no limit cycles in models of competitive or co-operative populations – Validate the statement via mathematical analysis. 3+3=6

2. a) The generalized Verhulst population model, with crowding effect of the form $\left(-\beta N^{\alpha}\right)$ is given by

$$\frac{dN}{dt} = rN - \beta N^{\alpha} \left(\alpha > 1\right).$$

Obtain the steady state solution and check for stability.
Also, solve the model for N and predict the behaviour of the population for a long period of time.
Find the point of inflexion, if any. 1+(2+1)+1=5

b) Consider the following predator-prey system with Holling-Tanner type II functional response

$$\frac{du}{dt} = ru\left(1 - \frac{u}{K}\right) - \frac{muv}{a+u},$$

$$\frac{dv}{dt} = \left(\frac{emu}{a+u} - d\right)v,$$

$$u(0) > 0,\ v(0) > 0,\ 0 < e < 1.$$

Find the parametric restrictions for which the co-existence equilibrium point $\left(u^*, v^*\right)$ is globally asymptotically stable with the use of the following Lyapunov function

$$L(u,v) = \int_{v^*}^{v} s^{\theta-1}\left(s - v^*\right) ds + cv^{\theta} \int_{u^*}^{u} \frac{\xi - u^*}{\xi} d\xi, \quad \text{for some } c > 0,\ \theta > 0.$$ 5

P.T.O.

3. Consider the following ratio-dependent Bazykin's interacting species model:

$$\frac{du}{dt} = au - \frac{buv}{v+Au} - eu^2,$$

$$\frac{dv}{dt} = -cv + \frac{duv}{v+Au} - hv^2,$$

$$u(0) = u_0 > 0,\ v(0) = v_0 > 0.$$

Write down your observations around the co-existence equilibrium point on the subsequent issues.
(i) Bonndedness, (ii) Global stability through Dulac's criteria,
(iii) Hopf-bifurcation, (iv) Saddle-node bifurcation. 2+2+3+3=10

4. a) Note down the influence of the refuge parameter 'm' and harvesting efforts E_1 and E_2 about the co-existence fixed point of the following interacting species system:

$$\frac{du}{dt} = \alpha u\left(1-\frac{u}{K}\right) - \frac{\beta(u-m)v}{1+a(u-m)} - q_1E_1u,$$

$$\frac{dv}{dt} = -dv + \frac{c\beta(u-m)v}{1+a(u-m)} - q_2E_2v,$$ 2+3=5

$$u(0) > 0,\ v(0) > 0,\ 0 < c < 1.$$

b) Comment on 'optimal harvesting policy' via 'Pontryagin's maximal principle' around unique positive equilibrium point of the following host-parasite model system:

$$\frac{dH}{dt} = (r_1 - b_1H)H - a_1HP - c_1H,$$

$$\frac{dP}{dt} = \left(r_2 - \frac{a_2P}{H}\right)P - c_2P,$$ 5

$$H(0) > 0,\ P(0) > 0,\ 0 \le c_i < r_i,\ i = 1,2.$$

5. a) Develop and analyze the SIR model for general epidemic and consequently comment on Kermack and McKendrick threshold theorem. 6

b) Consider the two compartment SIS model and assume that all infected individuals recover and immediately re-enter the susceptible class. Find the equilibria of the model and establish their stability in the context of R_0. 4

6. An eco-epidemiological interacting species model where predators distinguish between susceptible and infected prey is given by

$$\frac{dS}{dt} = rS\left(1-\frac{S+I}{K}\right) - \lambda SI - \frac{pYS}{mY+S},$$

$$\frac{dI}{dt} = \lambda SI - \frac{cYI}{mY+I} - \gamma I,$$

$$\frac{dY}{dt} = \delta Y\left(1-\frac{hY}{I+S}\right),$$

$$S(0) \ge 0,\ I(0) \ge 0,\ Y(0) \ge 0.$$

a) Determine and classify all the ecologically meaningful equilibria.

b) Can you interpret the impact of disease on prey in the absence of predators and disease effects on predators in the absence of healthy prey? 4+(3+3)=10
