M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMO-31 (A3) (New)

(Computational Fluid Dynamics-I)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt ***any four*** questions.

1. a) Classify the partial differential equation

$$\frac{\partial^2 u}{\partial t^2}+4\frac{\partial^2 u}{\partial t\partial x}+4\frac{\partial^2 u}{\partial x^2}+2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}=0$$

and find its characteristics. Reduce the equation to its standard form. 5

b) Find the fourth order accurate discretized form of first order partial derivative with respect to one independent variable using the values of the dependent variable at the $(i+2,j),(i+1,j),(i-1,j)$ and $(i-2,j)$ points. 5

2. a) Write down the Crank-Nicholson scheme for the numerical solution of one-dimensional heat conduction equation. Find the truncation error and stability criteria of the method. 1+2+2

b) Use the Crank-Nicholson method and the central differences for the boundary conditions to solve the initial boundary value problem

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0)=1,\ 0\le x\le 1$$

$$\frac{\partial u}{\partial x}(0,t)=u(0,t),\ \frac{\partial u}{\partial x}(1,t)=-u(1,t),\ t>0$$ 5

with step length $h=\frac{1}{2}$ and $\lambda=\frac{1}{3}$ Integrate for one time level. Here, $\lambda = k/h^2$, where h and k are step lengths in x and t directions, respectively.

3. a) Find the Peaceman-Rachford ADI method for the two dimensional heat flow equation in the domain $R=[0\le x,\ y\le 1]\times[0,T]$ subject to initial and boundary conditions. Also find the stability criteria of the method. 2+3

b) Consider the general heat equation with a complex coefficient

$$\frac{\partial u}{\partial t}=(a+ib)\frac{\partial^2 u}{\partial x^2}.$$ 5

Show that if $a\ge 0,$ the Crank-Nicolson method is unconditionally stable.

4. Derive Wendroff, Lax-Wendroff and Leap-Frog schemes for the solution of first order hyperbolic partial differential equation. Also, find their conditions of stability and truncation errors involved. 3+7

P.T.O.

5. Find the solution of the initial value problem

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$$

together with an initial condition

$$u=0, \quad x<0$$
$$=x/2 \quad 0\le x\le 2$$
$$=2-x/2 \quad 2\le x\le 4$$
$$=0 \quad x>4$$

Using (i) Lax-Wendroff method, (ii) Leap-Frog scheme with $h=0.5,\ r=0.5$ where $r={}^{k}\!/_{h}$, h and k being the step lengths in x and t directions, respectively. Compute upto two time stems. 5+5

6. Solve the partial differential equation

$$\nabla^2 u=x^2-1, \quad |x|\le 1, |y|\le 1$$

$u=0$ on the boundary of the square.

Formulate the five point and nine point difference schemes with mesh size h in both directions. Solve the difference equations for $h=\frac{1}{2}$. 10

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