Use separate answer script for each unit

M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMC-34 (New)

(Calculus of Variations and Special Functions)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings.

Unit-I (Marks: 20)

Answer ***any two*** questions.

1\. a) Show that a necessary condition for the functional

$$I=\int_{x_1}^{x_2} F\left(x,y,y',y''\right)dx$$

to be extremum is $F_y-\frac{d}{dx}F_{y'}+\frac{d^2}{dx^2}F_{y''}=0,$

where the 'dash' denotes differentiation with respect to x. 3

b) Find the stationary function of

$$I\left(y,z\right)=\int_0^{\pi/2}\left[\left(y'\right)^2+\left(z'\right)^2+2yz\right]dx$$

under the conditions $y\left(0\right)=3,\ y\left(\pi/2\right)=1,\ z\left(0\right)=-3,\ z\left(\pi/2\right)=-1.$ 3

c) Find the curve with fixed boundary points such that its rotation about the axis of abscissae gives rise to a surface of revolution of minimum surface area. 4

2\. a) Find the stationary function of

$$I\left(y\right)=\int_0^1\left[\tfrac{1}{2}\left(y''\right)^2-y\right]dx \quad \text{with } y\left(0\right)=0,\ y\left(1\right)=0.$$ 4

b) Show that sphere is the solid figure of revolution which, for a given surface area, has maximum volume. 4

c) Reduce the following boundary value problem into a variational problem:

$y''-y+x=0$ with $y\left(0\right)=y\left(1\right)=0$. 2

3\. a) Find the curves for which the function

$$I=\int_0^{x_1}\frac{\sqrt{1+y'^2}}{y}dx \quad \text{with } y\left(0\right)=0 \text{ can have extrema, if}$$ 4

i) The point $\left(x_1,y_1\right)$ can be vary along the circle $\left(x-9\right)^2+y^2=9,$

ii) The point $\left(x_1,y_1\right)$ can vary along the line $y=x-5$.

b) Describe Rayleigh-Ritz method for solving a boundary value problem. 2

c) Use Galerkin method to solve the boundary value problem

$y''-y+x=0\ \left(0\le x\le 1\right)$ with $y\left(0\right)=0,\ y\left(1\right)=0$. 4

Compare your approximate solution with the exact solution.

P.T.O.

Unit-II (Marks: 20)

Answer ***any two*** questions.

1. a) Discuss the singularities of the equation 2

$$x^2y'' + xy' + \left(x^2 - n^2\right)y = 0 \text{ at } x = 0 \text{ and } x = \infty .$$

b) Express $2 - 3x + 4x^2$ in terms of Legendre polynomials. 2

c) Solve the Bessel's equation for the regular singular point at the origin and hence obtain the Bessel's function of first kind of order n. 6

2. a) Show that Hermite polynomials are orthogonal over $\left(-\infty, \infty\right)$ with respect to the weight function e^{-x^2}. 5

b) Prove that 2×1=2

i) $e^x = {}_1F_1\left(\alpha; \alpha; x\right)$

ii) $\left(1-x\right)^{-x} = {}_2F_1\left(\alpha, \beta; \beta; x\right)$

c) Prove that

$$\frac{Exp\left\{-xt/\left(1-t\right)\right\}}{1-t} = \sum_{n=0}^{\infty} L_n\left(x\right)t^n .$$ 3

3. a) Considering a spring system hit by a hammar at $t = 0$, solve the Differential equation $x'' + x = \delta(t)$. 3

b) Prove that,

$$P_n\left(\cos\theta\right) = \frac{1.3.5....\left(2n-1\right)}{2^{n-1}.n!}\left[\cos n\theta + \frac{1.n}{1.\left(2n-1\right)}\cos\left(n-2\right)\theta + \frac{1.3....n\left(n-1\right)}{1.2....\left(2n-1\right)\left(2n-3\right)}\cos\left(n-4\right)\theta + ...\right]$$ 3

c) For $J_n\left(x\right)$, prove that 2×2=4

i) $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{n-1}(x)$

ii) $\frac{d}{dx}\left\{x^{-n} J_n(x)\right\} = x^{-n} J_{n+1}(x)$