

M.A./M.Sc. Examination 2018
Semester - III
Mathematics
Course: MMC-33 (New & Old)
(Fluid Mechanics)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt **any four** questions.

1. a) Show that the equation of continuity by the Eulerian method and Lagrangian method are equivalent. 3
b) Consider a two dimensional incompressible steady flow field with velocity components in spherical coordinates (r, θ, ϕ) given by
$$v_r = c_1 \left(1 - \frac{3 r_0}{2 r} + \frac{1 r_0^2}{2 r^2} \right) \cos \theta,$$
3
$$v_\theta = -c_1 \left(1 - \frac{3 r_0}{4 r} - \frac{1 r_0^2}{4 r^2} \right) \sin \theta, v_\phi = 0,$$
$$r \geq r_0 > 0$$
 where c_1 and r_0 are arbitrary constants. Is the equation of continuity satisfied?
c) If the velocity components for a two-dimensional fluid system are given in the Eulerian system by $u = 2x + 2y + 3t$, $v = x + y + \frac{t}{2}$, find the displacement of a fluid particle in the Lagrangian system. 4
2. a) Find the condition that the surface $F(x, y, z, t) = 0$ may be a boundary surface of a fluid in motion. Also find the normal velocity of the boundary. 2
b) Establish the relation connecting the angular velocity and the vorticity vector. 2
c) Show that if the motion of an ideal fluid, for which density is a function of pressure only, is steady and the external forces are conservative, then there exists a family of surfaces which contain the stream lines and vortex lines. 3
d) At a point in an incompressible fluid having spherical polar co-ordinates (r, θ, ϕ) , the velocity components are $(2Mr^{-3} \cos \theta, Mr^{-3} \sin \theta, 0)$, where M is a constant. Show the velocity is of the potential kind. Find the velocity potential and the equations of the stream lines. 3
3. a) Find the physical significance of stream function. 2
b) State and prove Milne-Thomson circle theorem. 3
c) Determine image system for a source outside a circle of radius a with the help of the circle theorem. 2

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- d) In the case of the two-dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O , show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the lines joining S to the ends of the diameter at right angles to OS meet the circle. Prove that its magnitude at these points is $(2m \times OS) / (OS^2 - a^2)$. 3
4. a) State and prove Kelvin's minimum energy theorem. 1+2
 b) A velocity field is given by $\vec{q} = (-\vec{i}y + \vec{j}x) / (x^2 + y^2)$. Determine whether the flow is irrotational. Calculate the circulation round a unit circle with centre at the origin. 2
 c) Determine the motion of a circular cylinder moving in an infinite mass of the liquid at rest at infinity, with velocity U in the direction of x -axis. Also find the equation of the stream lines. 4+1
5. a) Prove that the state of stress at a point is completely known if the nine components of stress tensor at that point are known. 3
 b) If in two dimensions, x axis and y axis are rotated through an angle θ , then find the expressions for new stress components in terms of old stress components. Also find the basic invariants for two-dimensional stress components. 4
 c) The stress matrix at a point P is given by $\begin{pmatrix} 2 & 1 & -3 \\ 1 & 1 & 2 \\ -3 & 2 & 1 \end{pmatrix}$. Find the stress vector on the plane passing through P and parallel to the plane whose unit normal is $\frac{3}{7}\vec{i} + \frac{6}{7}\vec{j} + \frac{2}{7}\vec{k}$. Also determine the principal stresses and principal directions for this stress tensor. 3
6. a) Derive the Navier-Stokes equation of motion for an incompressible viscous fluid assuming the constitutive equations for Newtonian fluid. 5
 b) Discuss the generalized Couette flow. Determine the volumetric flow rate, maximum and minimum velocities, shearing stress, skin friction and the coefficient of friction for this flow. 5
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