M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMC-32

(Solid Mechanics)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt ***any four*** questions.

1. a) Show that $dl^2 - dL^2 = 2E_{ij}dx_i dx_j$ where
$E_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right]$ is the Lagrangian finite strain tensor and dl, dL are the lengths of a small line vector element in the deformed and undeformed states respectively in a continuous medium while the coordinates refer to the undeformed state, and u_i is the displacement components. 5

   b) A displacement field is given by
   $$x_1 = X_1 + 2X_3$$
   $$x_2 = X_2 - 2X_3$$
   $$x_3 = X_3 - 2X_1 + 2X_2$$
   Determine Eulerian infinitesimal strain tensor, where (X_i) be the coordinates of a point in the undeformed state and (x_i) be the coordinates of the same point in the deformed state. 5

2. a) Prove that the maximum shearing strain at any point of the continuum is equal to one-half the difference between algebraically the largest and smallest principal strains at that point. 5

   b) The strain tensor at a point is given by
   $$e_{ij} = \begin{pmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{pmatrix}$$
   Calculate the principal strains and determine the shear between the directions $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$. 5

P.T.O.

3. a) Prove that the stress tensor is symmetric. 5

   b) The state of the stress at a point is given by

$$\tau_{ij} = \begin{pmatrix} T & aT & bT \\ aT & T & cT \\ bT & cT & T \end{pmatrix},$$ where $a,\ b,\ c$ are constants and T is some stress value.

   Determine the constants $a,\ b,\ c$ so that the stress vector on a plane normal to $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ vanishes. 5

4. a) Assuming symmetry of stress tensor, establish the relation

$$T_i^{\gamma} = \tau_{ij}\gamma_j \left(i, j = 1,2,3\right)$$ 5

   b) Show that the following stress components satisfy the equations of equilibrium with zero body forces but are not solutions of a problem in elasticity

$$T_{11} = \alpha\left[x_2^2 + \sigma\left(x_1^2 - x_2^2\right)\right],\ T_{22} = \alpha\left[x_1^2 + \sigma\left(x_2^2 - x_1^2\right)\right],$$

$$T_{33} = \alpha\sigma\left(x_1^2 + x_2^2\right),\ T_{12} = -2\alpha\sigma x_1 x_2,\ T_{23} = 0 = T_{31}.$$ 5

5. a) State the principle of balance of linear momentum and hence obtain Cauchy's first equation of motion. 1+4

   b) Prove that the strain energy density function is a positive definite quadratic form in the strain components. 5

6. a) State the second *BVP* of elastostatics and obtain the Navier's equation of equilibrium. 5

   b) Show that the Navier's equation of motion in absence of body forces $\left(\lambda + \mu\right)\Delta,\, i + \mu\nabla^2 u_i = \rho\ddot{u}_i$ is satisfied by $\vec{u} = \vec{\nabla}\varphi + curl\,\vec{\psi}$ provided φ and $\vec{\psi}$ satisfy three-dimensional wave equation. 5

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