B.A./B.Sc. (Honours) Examination 2018

Semester - V

Mathematics

Course: BME-51

(Numerical Analysis)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) Derive the expressions for the absolute and the relative errors of a function $f\left(x_1, x_2, ..., x_n\right)$ of n independent quantities whose individual errors are known. Hence obtain the relative error in $f = x_1^{m_1} x_2^{m_2} ......x_n^{m_n}$ in terms of the individuals relative errors of $x_1, x_2, ..., x_n$. 2+2

   b) Explain what do you mean by an approximate number and the significant figures of a computed number. What is the convention adopted in rounding off a number? Establish a rule for determining the propagated error for the addition operation. 1+1+2

   c) Show that $\Delta\begin{pmatrix} n \\ x+1 \end{pmatrix} = \begin{pmatrix} n \\ x \end{pmatrix}$, where the forward difference operator Δ operates on n and hence show that $\sum_{n=1}^{N}\begin{pmatrix} n \\ i \end{pmatrix} = \begin{pmatrix} N+1 \\ i+1 \end{pmatrix} - \begin{pmatrix} 1 \\ i+1 \end{pmatrix}$. 2

2. a) Derive an expression of the error involved in approximating a function by an interpolating polynomial when the functional values are known at $(n+1)$ distinct points. 4

   b) Carry out the differences of a polynomial of degree n and show that its nth order difference is a constant. 3

   c) By comparing the function $y = e\sin\left(\dfrac{x}{e^2}\right)$ with the polynomial approximation $L_n(x) = 0$, show that the accurate approximation of the function does not necessarily mean an accurate approximation of its derivative, where e is a small number. 3

3. a) Establish Newton-Cotes quadrature formula for numerical integration $\int_a^b f(x)dx$ where the function $f(x)$ is known at $(n+1)$ equi-spaced points.
   If $K_r^{(n)}$ be the Cotes' numbers, then show that (i) $\sum_{r=0}^{n} K_r^{(n)} = 1$, (ii) $K_r^{(n)} = K_{n-r}^{(n)}$ $\left(r = 0, 1, 2, ..., n\right)$. 3+1+2

   b) Define the degree of precision of a quadrature formula and hence find it for Weddle's rule of integration. Derive the error in Simpson's 1/3rd rule from Newton-Cotes quadrature formula. 1+1+2

P.T.O.

4. a) Show that the estimate of the error of the linear iteration method in the approximate value x_n of one real root of $f(x)=0$ is given by $|\xi - x_n| \le \frac{m}{1-m}|x_n - x_{n-1}|$, where ξ is a root of the equation and *m*, a positive proper fraction. 2

b) Explain the Regula-Falsi method to determine approximately one real root of an equation $f(x)=0$ and discus its convergence. 3+2

c) Write down the local and global errors of trapezoidal formula and state which one is smaller and why. 1+1+1

5. a) Explain the Newton-Raphson method for computing a simple real root of an equation $f(x)=0$. Show that this method has a quadratic convergence. 3+2

b) What do you mean by 'separation of roots'? Explain Graeffe's root-squaring technique to solve the general algebraic equation of degree *n* having a pair of complex roots and other distinct real roots. Is it possible to improve accuracy of the roots by repeating the process once the roots are already separated. 1+3+1

6. a) Explain briefly Gauss-Seidel iterative method for solving a system of *n* linear equations involving *n* unknowns represented by $\sum_{j=1}^{n} a_{ij}x_j = b_j,\ i=1,2,...,n$ with real coefficients. State a sufficient condition for convergence of the iterative scheme. 3+1

b) What is meant by 'pivotal condensation'? Carry out the total operational count required for solving a system of *n* linear equations by Gauss elimination method. 1+3

c) Examine the convergence of the Jacobi's iterative process for solving a system of linear equations. 2

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