B.A./B.Sc. (Honours) Examination 2018

Semester - V

Mathematics

Course: BMC-54

(Mechanics-III)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) If the moments and products of inertia of a rigid body about three mutually perpendicular axes $OXYZ$ be known, then find the moment of inertia about any other axis *OQ* through their meeting point. Hence find the momental ellipsoid of a body about the origin *O*. 6

   b) The lengths *AB* and *AD* of the sides of a rectangle *ABCD* are *2a* and *2b*; show that the inclination to *AB* of one of the principal axes at *A* is $\frac{1}{2}\tan^{-1}\frac{3ab}{2\left(a^2-b^2\right)}$. 4

2. a) State and prove the perpendicular axes theorem for a rigid body. 4

   b) Explain when two systems are said to be *equimomental*. Show that the moments of inertia of a uniform triangle *ABC* about any lines are the same as the moments of inertia about the same lines, of three particles placed at the middle points of the sides, each equal to one-third of the mass of the triangle. 6

3. a) State D'Alembert's principle for a rigid body motion and hence deduce the equations of motion. 5

   b) A uniform rod *OA* of length, *2a*, free to turn about its end *O*, revolves with uniform angular velocity $\vec{\omega}$ about the vertical *OZ* through *O*. If the rod describes a cone of semi-vertical angle α to *OZ*, then show that, $\omega^2 = \frac{3g}{4a\cos\alpha}$ and the resultant reaction of the rod at *O* makes an angle, $\phi = \tan^{-1}\left(\frac{3}{4}\tan\alpha\right)$, with the vertical component. 5

4. a) Prove that, $\vec{\Lambda} = \frac{d\vec{\Omega}}{dt}$, where $\vec{\Lambda}$ is the net moment of the force applied on a rigid body and $\vec{\Omega}$ is the total angular momentum (or moment of momentum) about the origin *O*. 5

   b) Prove that, the total angular momentum of a rigid body about any point *O* equals to the angular momentum of the total mass assumed to be located at the centre of mass *C* plus the angular momentum about the centre of mass:
   $$\vec{\Omega} = \vec{\Omega}_C + \vec{\Omega}'.$$ 3

   c) Prove that the total *angular impulse* is equal to the change in the *total angular momentum*. 2

P.T.O.

5. a) A ladder of length l and weight W_l has one end against a vertical wall which is frictionless and the other end on the ground assumed horizontal. The ladder makes an angle α with the ground. Prove that a man of weight W_m will be able to climb the ladder without having it slip if the coefficient of friction μ between the ladder and the ground is at least $\dfrac{W_m + \frac{1}{2}W_l}{W_m + W_l}\cot\alpha$. 5

b) A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position; show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle of 45° to the vertical, and that the vertical strain is then $\dfrac{11}{8}$ times the weight of the rod. 5

6. a) Prove that, for motion in two dimensions, the kinetic energy of a body of total mass M can be expressed as $T = \frac{1}{2}Mv_C^2 + \frac{1}{2}Mk^2\left(\frac{d\theta}{dt}\right)^2$, where $\vec{v}_C$ is the velocity of the centre of gravity C, θ is the angle that any line fixed in the body makes with a line fixed in space, and k is the radius of gyration of the body about a line through C perpendicular to the plane of motion. 5

b) A uniform solid sphere rolls down an inclined plane of angle, α, rough enough to prevent any sliding. Show that the centre of the sphere moves with a constant acceleration $= \frac{5}{7}g\sin\alpha$. Find also its velocity and the distance it has travelled in time t. 5

———