B.Sc. (Honours) Examination 2018

Semester – III (CBCS)
Mathematics
Course: CC-7
(Differential Equations-I)

Time: Three Hours | Full Marks: 60

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer question no. **1** and ***four*** from the rest.

1. Answer ***any ten*** questions: 10×2=20

   i) Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx}=(y-1)(y-5)(y-7)$ satisfying the initial condition $y(0)=6$.
   Find the value of $y(x)$ when $x\to\alpha$.

   ii) If an integral curve of the differential equation $(y-x)\frac{dy}{dx}=1$ passes through the points $(0,0)$ and $(\alpha,1)$, then find the value of α.

   iii) Find the differential equation of all parabolas having their axes parallel to the y-axis.

   iv) Find the orthogonal trajectories of the family of curves $y=cx^3$, c being a parameter.

   v) Let $y(x)$ be the solution of the differential equation $\frac{d}{dx}\left(x\frac{dy}{dx}\right)=x,\ y(1)=0,\ y'(1)=0$. What is the value of $y(5)$?

   vi) Let $g(x,y)dx+(x+y)dy=0$ is an exact differential equation where $g(x,0)=x^2$. Find the general solution of the above differential equation.

   vii) Consider the differential equation $x^2y''-x(x+2)y'+(x+2)y=x^3$. If $y=x$ is a linearly independent solution of the above differential equation, then find a second linearly independent solution.

   viii) If $y(t)$ is a solution of the differential equation $y''+4y=2e^t$, then evaluate $\lim_{t\to\infty}e^{-t}y(t)$.

   ix) Find an integrating factor of the differential equation $\sqrt{x}\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+3y=0$.

   x) Solve : $\left(\frac{dy}{dx}\right)^3-(x^2+xy+y^2)\frac{dy}{dx}+x^2y+xy^2=0$.

   xi) A body whose temperature is initially $100^o c$ is allowed to cool in air, whose temperature remains at a constant temperature $20^o c$. It is given that after 10 minutes, the body has cooled to $80^o c$. Find the temperature of the body after half an hour.

P.T.O.

xii) Let $y_1(x)$ and $y_2(x)$ be two solutions of $\left(1-x^2\right)y''-2xy'+\left(secx\right)y=0$ with Wronskian $W(x)$. If $y_1(0)=1,\ y_1'(0)=0$ and $W\left(\frac{1}{2}\right)=\frac{1}{5}$, then find the value of $y_2'(0)$.

xiii) Find four different solutions of the initial value problem. $\frac{dy}{dx}=xy^{\frac{1}{2}},\ y(0)=0$.

2. a) Solve the differential equation

$$\frac{dy}{dx}+y=g(x),\ y(0)=2,\ 0\le x<\infty \text{ where } g(x)=\begin{cases}3, & 0\le x<\frac{\pi}{2}\\ \cos x, & x\ge\frac{\pi}{2}\end{cases}$$ 5

b) Prove that the transformation $y=f+\frac{1}{v}$ reduces the differential equation $\frac{dy}{dx}=A(x)y^2+B(x)y+C(x)$ to a linear equation in *v*, where *f* is any solution of the above differential equation.
Hence solve the equation

$$\frac{dy}{dx}=-8xy^2+4x\left(4x+1\right)y-\left(8x^3+4x^2-1\right);\text{ given that } f(x)=x.$$ 2+3

3. a) Find a family of oblique trajectories that intersect the family of parabolas $y^2=cx$ at angle 60^o. 4

b) Find the orthogonal trajectories of the family of curves $r^n=a^n\cos n\theta$, a being a parameter. 2

c) Solve: $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\frac{1}{4}\left(x+xy^2\right)dy=0.$ 4

4. a) Solve: $\left(\frac{dy}{dx}\right)^3-4xy\left(\frac{dy}{dx}\right)+8y^2=0.$ 4

b) If $x^m y^n$ is an integrating factor of the differential equation $\left(x^7y^2+3y\right)dx+\left(3x^8y-x\right)dy=0$, then find the value of *m* and *n*. 2

c) Solve, by the method of variation of parameters:

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{1}{x^2e^x}$$ 4

5. a) Examine for singular solution and extraneous loci, if any, of the differential equation

$$\left(\frac{dy}{dx}\right)^2\left(2-3y\right)^2=4\left(1-y\right).$$ 5

b) Solve: $\frac{d^2y}{dx^2}-y=x\sin x+\left(1+x^2\right)e^x.$ 4

c) What is difference between the function $y=\frac{1}{x}$ and solution of the differential equation $xy'+y=0,\ y\left(1\right)=1$. 1

6. a) Reduce the differential equation

$$\left(\frac{dy}{dx}x^2+y^2\right)\left(x\frac{dy}{dx}+y\right)=\left(\frac{dy}{dx}+1\right)^2$$

to Clairaut's form by using the substitutions $u=xy$ and $v=x+y$ and hence find its complete primitive. 5

b) Solve the equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=10e^{2x}-18e^{3x}-6x-11,$$

by the method of undetermined coefficients. 5

7. a) Verify that the differential equation

$$\left(1+x+x^2\right)\frac{d^3y}{dx^3}+\left(3+6x\right)\frac{d^2y}{dx^2}+6\frac{dy}{dx}=0 \text{ is exact.}$$ 2

b) Solve: $y^3\frac{d^2y}{dx^2}=5.$ 3

c) Solve: $x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0.$ 2

d) If $e^{\int\phi\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right)}$ is an integrating factor of the differential equation $M\left(x,y\right)dx+N\left(x,y\right)dy=0$, then find the function $\phi\left(\frac{x}{y}\right)$. 3