# B.Sc. (Honours) Examination 2018

**Semester – I (CBCS)**

**Mathematics**

**Course: CC-2**

**( Algebra-I )**

Time: Three Hours | Full Marks: 60

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any six*** questions.

1. a) Draw the column picture for the solution of the system of equations $x+y=5,\ x-y=-1$. 2

   b) Show that the following system of equations is inconsistent.
$$y-4z=8$$
$$2x-3y+2z=1$$
$$4x-8y+12z=1$$
2

   c) Find an equation involving $g,h,k$ that makes the following augmented matrix corresponds to a consistent system
$$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{pmatrix}.$$
2

   d) Find the reduced row echelon form of
$$\begin{pmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{pmatrix}.$$
Hence or otherwise list the free and basic variables of the corresponding system of equations. Also find its solution. 4

2. a) Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent. 3

   b) Find the equation to describe geometrically, span $\left(\{(1,2,1),\ (2,-3,0)\}\right)$. 2

   c) Express $b=(7,4,-3)$ as a linear combination of $a_1=(1,-2,-5)$ and $a_2=(2,5,6)$. 2

   d) Suppose $A$ is a $3\times 3$ matrix and $b$ is a vector in $\mathbb{R}^3$ with the property that $Ax=b$ has a unique solutions. Explain why the columns of $A$ must span $\mathbb{R}^3$. 3

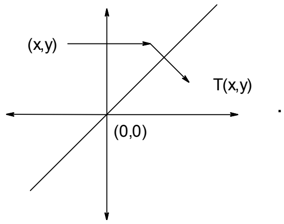
3. a) Examine if the columns of the matrix $\begin{pmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{pmatrix}$ are linearly independent. 3

   b) Prove that a set of vectors $S=\{v_1,v_2,...,v_p\}$ in $\mathbb{R}^n$ is linearly dependent if and only if at least one of the vectors is $S$ is a linear combination of the others. 4

**P.T.O.**

c) Suppose that an $m \times n$ matrix $A$ has $n$ pivot columns. Explain why for each $b$ in $\mathbb{R}^m$ the system $Ax = b$ has at most one solution. 3

4. a) Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ which represents the following two consecutive reflections. 2




b) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by $T(e_1) = (2, -3, 1)$, $T(e_2) = (0, -2, 5)$ and $T(e_3) = (-1, 3, 4)$. Find $T(x, y, z)$, where $e_1$, $e_2$ and $e_3$ are the standard basis vectors of $\mathbb{R}^3$. Hence find the standard matrix of $T$. 2+2

c) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and $A$ be the standard matrix of $T$. Prove that $T$ maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of $A$ spans $\mathbb{R}^m$. 4

5. a) Let $A$ be an $m \times n$ matrix. If $A$ is invertible, then show that the linear transformation $x \to Ax$ is one-to-one. 3

b) Find a basis for the null space of the matrix $A = \begin{pmatrix} -3 & 6 & 1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$. 3

c) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Show that $\ker T$ is a subspace of $\mathbb{R}^n$. 2

d) What can you say about null $B$, when $B$ is a 5×4 matrix with linearly independent columns. 2

6. a) Determine the rank of the matrix

$$A = \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{pmatrix}$$ 2

b) Find a basis and the dimension of $W$ where $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + 2y + 3z = 0\}$ 3

c) Find the eigen values of

$$A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$$

Also find the eigen space corresponding to the repeated eigen value, if any. 3

d) Construct an example of a 2×2 matrix which has only one eigen value. 2

7. a) Let $z_1$ and $z_2$ be two non zero complex numbers. If $\theta_1$ and $\theta_2$ be arguments of $z_1$ and $z_2$ respectively, then show that $\theta_1 + \theta_2$ is an argument of $z_1 z_2$.

Does the result hold it one considers the principal arguments of complex numbers? Justify your answer. 2+2

b) If $n$ is a positive integer and $(1+z)^n = a_0 + a_1 z + \ldots + a_n z^n$, then prove that

(i) $a_0 - a_2 + a_4 - \ldots = 2^{n/2} \cos n\pi/4$

(ii) $a_1 - a_3 + a_3 - \ldots = 2^{n/2} \sin n\pi/4$ 3

c) Show that the product of all values of $\left(\sqrt{3}+i\right)^{3/5}$ is $8i$. 3

8. a) Let $a, b, c, d$ be four positive real numbers, not all equal. Then show that the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 + (x-d)^3 = 0$ has only one real root. 4

b) If $\alpha, \beta$ and $\gamma$ be the roots of the equation $2x^3 + x^2 + x + 1 = 0$, find the equation whose roots are $\frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}, \frac{1}{\alpha^2} - \frac{1}{\beta^2} + \frac{1}{\gamma^2}, -\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. Hence find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. 3+1

c) Expand $f(x) = x^4 - x^3 + 2x^2 + 6x - 2$ as a polynomial in $x-2$ and then find $f(x+2)$ 2

9. a) If $x, y, z$ are positive real numbers and $x+y+z=1$, prove that

$$8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}.$$ 3

b) Solve $x^3 + 6x^2 - 12x + 32 = 0$. 4

c) Express $f(x) = x^4 - 2x^3 - 5x^2 + 10x - 3$ in the form $\left(x^2 + px + q\right)^2 - (ax+b)^2$ and hence, solve the equation $f(x) = 0$. 3

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