M.Sc. Examination, 2018
Semester – III
Physics
Course: MPC-355
(Quantum Field Theory)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Symbols have their usual meanings unless specified otherwise.
Answer ***any four*** of the following questions.

1. (a) 'The interpretation of Klein-Gordon equation as describing a relativistic quantum mechanical system is problematic'-verify the truth in the statement with adequate mathematical derivations, whenever necessary. 4

   (b) Discuss the reasons for requiring a symmetric energy-momentum tensor and suggest a possible way to convert a non-symmetric energy-momentum tensor into a symmetric one without changing the physical data. 1.5+1.5

   (c) What do you mean by normal ordering of operators? Discuss its relevance in the context of canonical quantization of fields. 3

2. Consider the Lagrangian density of a spinor field Ψ in $3+1$ dimensions:

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\Psi + g\left(\bar{\Psi}\Psi\right)^{2}$$

   (a) Obtain the equations of motion. 4

   (b) Obtain the Hamiltonian corresponding to $\mathcal{L}$. 3

   (c) Find the mass dimension of g in natural units and determine whether or not the theory is power-counting renormalizable. 3

3. Consider the Lagrangian density of a free complex scalar field:

$$\mathcal{L} = \left(\partial_{\mu}\phi\right)^{*}\left(\partial^{\mu}\phi\right) - m^{2}\phi^{*}\phi$$

   (a) Identify the internal symmetry and find an expression for the corresponding conserved charge Q. 1+2

   (b) The field ϕ can be decomposed in terms of Fourier modes as,

$$\phi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\frac{d^{3}k}{\sqrt{2k_{0}}}\left[e^{-ik\cdot x}A(\vec{k}) + e^{+ik\cdot x}B^{\dagger}(\vec{k})\right]$$

   Find an expression of the charge-operator $\hat{Q}$ corresponding to classical charge Q in terms of $A(k)$, $B(k)$ and their adjoints. *You may assume the commutation relations satisfied by $A(k), B(k)$ and their adjoints. Derivation of such relations are not required.* 4

   (c) The field $\phi(t,\vec{r})$ transforms under the charge conjugation operator C as follows: $C\phi(t,\mathbf{r})C^{-1} = \eta_{C}\phi^{\dagger}(t,\mathbf{r})$, where η_{C} is a phase factor. Determine how the charge-operator $\hat{Q}$ behaves under the action of the charge-conjugation operator C. 3

4. (a) Discuss the idea of gauge-fixing and its relevance in the context of quantization of free electromagnetic field. 3

   (b) Show that one photon state $|k,0\rangle$ in the covariant quantization scheme has negative norm, where $|k,0\rangle = a_{0}^{\dagger}(k)|\,0\rangle$. 2

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(c) Find a condition, with proper explanation and justification, so that the states in the covariant quantization scheme have semi-positive definite norm. 3

(d) Determine whether the state $|\psi\rangle$ in the covariant gauge is a physical state or not: 2

$$|\psi\rangle = (a_0^\dagger + a_3^\dagger)^2 |0, 0, 0, 0\rangle$$

5. The spinor field $\psi(x)$ may be decomposed into its Fourier moments with the following expression:

$$\psi(x) = \int \frac{d^3k}{(\sqrt{2\pi})^3} \sqrt{\frac{m}{k_0}} \sum_{\alpha=1,2} \left[c_\alpha(k) u_\alpha(k) e^{-ik\cdot x} + d_\alpha^\dagger(k) v_\alpha(k) e^{ik\cdot x} \right]$$

(a) Assuming the orthonormality relations among the spinors u_α and v_α, show that $c_\alpha(k)$ and $d_\alpha(k)$ can be determined as follows: 2+2

$$c_\alpha(k) = \int d^3x \sqrt{\frac{m}{k_0(2\pi)^3}} u_\alpha^\dagger(k) \psi(x) e^{ik\cdot x},$$

$$d_\alpha(k) = \int d^3x \sqrt{\frac{m}{k_0(2\pi)^3}} \psi^\dagger(x) v_\alpha(k) e^{ik\cdot x}$$

(b) State the equal-time anti-commutation relations satisfied by $c_\alpha(k)$, $d_\alpha(k)$ and their adjoints in the quantized theory. 2.

(c) Show that the normal ordered form of the momentum has the following expression: 4

$$\vec{p} = \int d^3k \; \vec{k} \sum_{\alpha=1,2} \left[c_\alpha^\dagger(k) c_\alpha(k) + d_\alpha^\dagger(k) d_\alpha(k) \right]$$

6. (a) Define $Q = \gamma_\mu \left(\not{p}_1 + m \right) \gamma_\nu \left(\not{p}_1' + m \right)$ and prove the identity: 4

$$\sum_{s,s'=1,2} \bar{u}^{(s')}(p_1') \gamma_\mu u^{(s)}(p_1) \bar{u}^{(s)}(p_1) \gamma_\nu u^{(s')}(p_1') = Tr(Q)$$

(b) Derive an expression for the field-strength of a non-abelian gauge-field with $SU(2)$ symmetry. 4

(c) What new features do you get for the non-abelian $SU(2)$ gauge-fields compared to the abelian $U(1)$ gauge-field? 2

7. (a) Write down the vertex contribution to the Feynman diagrams corresponding to the interaction Hamiltonian $H_I = \frac{\lambda}{4!}\phi^4$ of a real scalar field ϕ. 2

(b) Verify the truth in the statement: 'No physical process is described by any of the first order Feynman diagrams in quantum electrodynamics.' 2

(c) Draw second order Feynman diagrams and write down the corresponding scattering amplitude by applying Feynman rules for the QED process: $e^+ + e^+ \to e^+ + e^+$. 4

(d) Draw the vacuum polarization diagram of QED, determine its superficial degree of divergence and comment whether the corresponding amplitude is divergent or not. 2