M.Sc. Examination 2018
Semester – III
Physics
Course: MPC-31
(Statistical Mechanics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***Question No. 1*** and ***any three*** from rest of the questions.

1. Answer any *five* questions. **2 × 5**

   (a) If two systems of fixed volume are allowed to exchange the number of particles and energy, show that at equilibrium chemical potential is same for both the systems.

   (b) The restoring force of an anharmonic oscillator is proportional to the cube of the displacement. Show that the mean kinetic energy of the oscillator is twice its mean potential energy.

   (c) A system has non-degenerate energy levels with energy $(n+\frac{1}{2})\hbar\omega$, with $n = 0, 1, 2, ..$ and $\hbar\omega = 8.625 \times 10^{-5}$ eV. If the system is in contact with a heat bath at room temperature ($T = 300$ K) what is the probability that the system is in the state $n = 10$?

   (d) Given a system of two distinct lattice sites, each occupied by an atom whose spin ($s = 1$) so oriented that its energy takes one of the three values $\epsilon = 1, 0, -1$. The state with energy 1 have no degeneracy while states with energy -1 and 0 having doubly and triply degeneracy respectively. If we assume that atoms are non-interacting then find the average energy of the system.

   (e) In the classical limit, both the Bose-Einstein and Fermi-Dirac distribution functions are approximated to $n(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)}}$. For a three dimensional system of classical ideal gas of volume V at a temperature T show that $n\lambda^3 = z$, where n is the density of particles, z is the fugacity and λ is the thermal wavelength.

   (f) Consider a particle of mass m is initially at rest at a height h above a reference plane with zero potential energy. Draw the phase space diagram if it falls freely due to gravity.

   (g) If the energy of a classical one dimensional simple harmonic oscillator, with characteristic frequency ω, is allowed to vary between $E - \frac{1}{2}\Delta$ to $E + \frac{1}{2}\Delta$, where $\Delta << E$, show that the phase space area for the representative points is given by $\frac{2\pi}{\omega}\Delta$.

2. A classical ideal gas contains N number of indistinguishable, non-interacting particles in volume V at a temperature T.

   (a) Obtain the canonical partition function of the system. **3**

   (b) Hence, obtain entropy and the equation of state for the system. **3**

   (c) Show that relative energy fluctuation of the system vanishes in the limit of large N. **4**

P.T.O.

3. (a) State and prove the equipartition theorem by considering the canonical ensemble of a system of classical ideal gas characteraised by Hamiltonian $H(q,p)$. **6**

   (b) Find the density of states and total number of states of a free particle of mass 10^{-30} kg in a 3-dimensional cubical box of dimension 1 m^3 at room temperature ($T = 300\mathrm{K}$). **4**

4. (a) Consider the l-th level rotational energy of a diatomic molecule with moment of inertial I is given by $\epsilon_l = l(l+1)\hbar^2/2I$, where $l = 0, 1, 2, \ldots$. Find the rotational contribution towards specific heat at constant volume both at low and high temperature limit. **5**

   (b) For a system of classical ideal gas briefly discuss the phenomena of Gibbs paradox and how to resolve it. **5**

5. Suppose an ideal Bose gas contains N number of particles in volume V at a temperature T. The Bose-Einstein distribution function is given by $< n_\epsilon >= \frac{1}{z^{-1}e^{\beta\epsilon}-1}$, where, z is the fugacity and other symbols have their usual meanings.

   (a) Express pressure and internal energy of the system in terms of Bose-Einstein function, defined as $g_n(z) = \int_0^\infty \frac{x^{n-1}}{z^{-1}e^x-1}dx$. Hence, show that the internal energy, volume and pressure of the gas are related as $P = \frac{2}{3}\frac{U}{V}$. **4+1**

   (b) Obtain an expression for the critical temperature T_c, for fixed N and V, for the Bose-Einstein condensation. **5**

6. (a) What are the basic differences of Einstein and Debye specific heat models for solid? **2**

   (b) Show that in Debye model, at low temperature specific heat follows the T^3 law. **7**

   (c) What would be the value of specific heat of solid at high temperature? **1**

7. (a) Suppose an external magnetic field $\vec{B}$ is applied to a gas of volume V containing N number of spin-$\frac{1}{2}$ particles each with mass m and intrinsic magnetic moment $\vec{\mu}$. At absolute zero, obtain an expression for the low field susceptibility as a function of the fermi energy of the system. **3**

   (b) Using the Fermi-Dirac statistics discuss the statistical equilibrium of an ideal white dwarf star and obtain the relation between the mass and radius of the star. **7**
