B.A. (Honours) Examination, 2014

Semester - III

Integrated Mathematics & Statistics (Subsidiary)

Paper : S –1.5

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** from the following.

1. (a) Evaluate $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$ 4

   (b) Show that the function $f(x)$ defined by

   $$f(x)=3+2x \text{ for } -\frac{3}{2}<x\le 0$$
   $$=3-2x \text{ for } 0<x\le \frac{3}{2}$$

   In continuous at $x=0$ but not differentiable at $x=0$. 6

2. Find the deviates of

   (a) $\frac{x^3}{x+1}(w.r.t.x)$ (b) $\sin(10t)\,(w.r.t.\,t)$

   (b) Find the area of the triangle formed by the x. and y-*axex* and the tangent to the graph $y=\frac{1}{x}$ 2+2+6

3. (a) Obtain the Taylor's seris expression for $f(x+h)$

   (b) Find the quadratic approximations to

   (i) $\ln(1+x)$ (ii) e^x 4+3+3

4. (a) State and prove Euler's Theorem on homogeneous function of x and y of degree n. 5

   (b) If $\gamma=\tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{d\gamma}{dx}+y\frac{d\gamma}{dy}=\sin 24$. 5

5. (a) (i) Integrate $\int\frac{dx}{x\sqrt{(x^2-a^2)}}$

   (ii) Integrate $\int x^3e^x dx$ 3+3

   (b) Assuming the convergence of the integral $\int_0^{\pi/2}\log\sin x\,dx$, show that $\int_0^{\pi/2}\log\sin x\,dx=\frac{\pi}{2}\log\frac{1}{2}$ 4

P.T.O.

6. (i) Evaluate using integration by parts

(a) $\int \ln x \, dx$

(b) $\int x^n e^x \, dx$

(ii) Use l 'Hospital's rule to evaluate

$$\lim_{x \to 0} \frac{x^{10} - l}{x^2 - 1}$$

3+3+4

7. (a) Show that if a function $f(x_1 \ldots\ldots x_n)$ is homogenous of degree r then $\frac{\partial f}{\partial x_i}$ is homogeneous of degree $r-1$

(b) Consider the function

$$f(x_1, x_2) = x_1^3 x_2^{1/2} + x_2$$

is (a) this function concave or convex?

(b) Find the elasticity of scale of this function. 4+3+3

8. Consider the function

$$Q = x_1^{1/2} x_2^{1/3}$$

(a) is this function concave or convex

(b) maximize this function subject to $h_1 x_1 + h_2 x_2 = C$ 10
