B.A. (Honours) Examination, 2014

Semester - III

Integrated Mathematics & Statistics (Subsidiary)

Paper : S –1.4

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** from the following.

1. (a) Geometrically present the operation of vector addition and scalar multiplication.

   (b) Draw a plane and show the path you would traverse were you to start at (–1, –3) displace yourself first by the vector (1,–3) and then by the vector (–1,–3)

   (c) Find x,y,z such that (x–y, x+y, z–1)=(4,2,3) 4+4+2=10

2. (a) If P (a,b,c) and Q (a_2, b_2, c_2) are two vectors in R^3. Then find the length of the vector $\overrightarrow{PQ}$ applying Pythagorean Theorem.

   (b) Use vector notation to prove that the diagonals of a rhombus are orthogonal to each other.

   (c) For the following pairs of vectors, first determine whether the angle between them is acute or obtuse or right and then calculate the angle.

   (i) u=(1,0). v (2,2)

   (ii) u = (1,1,0) v (1,2,1) 5+3+2=10

3. (a) Define Linear dependence and Linear independence

   (b) Check whether (1,1,0), (0,1,1) & (1,0,1) – these vectors are linearly dependent or independent

   (c) What is parametric representation of a line? Write parametric equation of the line through P_1 (3,0) & P_2 (5,0). 2+4+4=10

4. Prove without expanding.

   (a) $$\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix} = 0$$

   (b) Find the inverse of the following matrix:

   $$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$$

(c) Express $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ as the product of elementary matrices. 3+4+3=10

5. Find the rank of the following matrix:

(a) $\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$

(b) Show that every square matrix can be expressed as a sum of a symmetric and a skew-symmetric matrix of same order. 5+5=10

6. Prove without expanding the determinant

(a) $\begin{vmatrix} (a+b)^2 & ca & bc \\ ca & (b+c)^2 & ab \\ bc & ab & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(b) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ 5+5=10

7. (a) Solve by Cramer's Rule

$-x_1 + 3x_3 + 1 = 0$

$2x_1 - x_2 - 4x_3 - 2 = 0$

$x_2 + 2x_3 - 4 = 0$

(b) Show that $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ is a perfect square. 5+5=10

8. (a) State Cayley-Hamilton theorem. Prove that similar matrices have the same characteristic polynomial. 1+4

(b) Use Cayley-Hamilton theorem to find A^{-1}, where $A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$. 2

(c) Find the eigen values and eigen vectors of the following matrix $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$. 3