

M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMC-32 (Old)

(Dynamical Systems)

(For Back Candidates)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings.

Attempt **any four** questions.

1. a) Obtain all critical points of the system $\dot{x} = \sin y$, $\dot{y} = \cos x$. Linearize the system about $(\pi/2, 0)$. Find the equation of the path. 1+1+1=3

- b) Show that for the following system

$$\dot{x} = -y + px(x^2 + y^2),$$

$$\dot{y} = x + py(x^2 + y^2),$$

(p is a parameter), the linearized system predicts the fixed point as a centre $\forall p$ but that point is a stable spiral if $p < 0$ and unstable spiral if $p > 0$. 4

- c) What is a flow? With the concept of flow, discuss the local stability of the fixed points of

$$\dot{x} = f(x)$$

$$= x - x^3.$$

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2. a) Using the Lyapunov function $L(x, y, z) = x^2 + 2y^2 + z^2$, show that the origin is an asymptotically stable equilibrium point of the system:

$$\dot{x} = -2y + yz - x^3,$$

$$\dot{y} = x - xz - y^3,$$

$$\dot{z} = xy - z^3.$$

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- b) Discuss the bifurcation of the following 2-dimensional systems: 3+3=6

i) $\dot{x} = \mu x - x^2,$

$$\dot{y} = -y,$$

and

ii) $\dot{x} = \mu x - x^3,$

$$\dot{y} = -y.$$

P.T.O.

3. a) Use Dulac criterion to show that the system $\dot{x} = x(\alpha - ax - by)$, $\dot{y} = y(\beta - cx - dy)$, where $a, d > 0$ has no closed orbit in the positive quadrant of \mathbb{R}^2 . 3

b) Show that the following system

$$\dot{x} = y + \frac{x}{\sqrt{x^2 + y^2}} \{1 - (x^2 + y^2)\},$$

$$\dot{y} = -x + \frac{y}{\sqrt{x^2 + y^2}} \{1 - (x^2 + y^2)\},$$

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has a stable limit cycle.

- c) State the Lienard theorem. Hence show that the equation $\ddot{x} + \mu(x^4 - 1)\dot{x} + x = 0$ has a stable limit cycle if $\mu > 0$. 1+3=4

4. a) Find all periodic two-orbit of the map $f(x) = 4x(1-x)$, $x \in [0, 1]$. Show that they are unstable. 2+3=5

- b) Define a tent map $T(x)$. Find $T^2(x)$. Show that $\left\{\frac{2}{7}, \frac{4}{7}, \frac{6}{7}\right\}$ and $\left\{\frac{2}{9}, \frac{4}{9}, \frac{8}{9}\right\}$ form two period-3 cycle of the tent map. 2+3=5

5. a) Define Semi conjugacy and conjugacy of a map. Show that the tent map

$$T(x) = 1 + 2x, \quad -1 \leq x \leq 0,$$

$$= 1 - 2x, \quad 0 \leq x \leq 1$$

2+3=5

is topologically conjugate to the map $\varphi_2(x) = 1 - 2x^2$, $-1 \leq x \leq 1$ through $h(x) = \sin \frac{\pi x}{2}$, $x \in [-1, 1]$.

- b) When is a fixed point called super stable? Show that +1 and -1 are super stable fixed points of the map $f(x) = \frac{3x - x^3}{2}$, $x \in \mathbb{R}$. 1+2=3

- c) Use Schwarzian derivative to obtain the stability of the fixed point at the origin of the function $f(x) = -\sin x$. 2

6. a) When is an 1-dimensional map said to be chaotic? Show that the doubling map $g : S \rightarrow S$ defined by $g(\theta) = 2\theta$, $\theta \in S$ is chaotic on the unit circle S . 1+4=5

b) Define Lyapunov exponent λ of a map. Show that λ can be expressed as

$$\lambda = \lim_{N \rightarrow \infty} \frac{1}{N} \ln \left| (f^N)'(x_0) \right|$$

Also, find the Lyapunov exponent for the tent map.

1+2+2=5