M.A./M.Sc. Examination 2018

Semester - III

Mathematics

Course: MMC-31 (Old & New)

(Discrete Mathematics)

(For Regular & Back Candidates)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Attempt ***any four*** questions.

1. a) Establish the validity of 3

$$(A \vee B) \to \big((C \vee D) \to E\big);$$
$$\therefore A \to \big((C \wedge D) \to E\big).$$

Construct a proof of invalidity of

$$T \equiv U$$
$$U \equiv (V \wedge W)$$
$$V \equiv (T \vee X) \qquad 3$$
$$T \vee X$$
$$\overline{}$$
$$\therefore T \wedge X$$

b) Establish the validity / invalidity of the argument:
Only citizens are voters. Not all citizens are residents. Therefore, some voters are not residents. 4

2. a) Show that
$(\forall x)F(x) \wedge (\forall x)G(x) \equiv (\forall x)\big(F(x) \wedge G(x)\big)$ is a valid biconditional whereas
$(\forall x)F(x) \vee (\forall x)G(x) \to (\forall x)\big(F(x) \vee G(x)\big)$ is conditional truth. 4

b) Construct a formal proof of validity for the argument:
If any jewellery is missing, then if all servants are honest, it will be returned. If any servant is honest, they all are. So if any jewellery is missing, then if at least one servant is honest, it will be returned. 4

c) Symbolize the following:
If any husband is unsuccessful, then if some wives are ambitious, he will be unhappy. 2

3. a) Define Hamiltonian graph. Let G be a graph with n vertices, and let u, v be non-adjacent vertices in G such that $d(u)+d(v)\geq n$. Show that G is Hamiltonian if and only if $G+uv$ is Hamiltonian. 1+2=3

b) Write algorithm for Depth-First-Search Spanning tree. 2

c) Define planar graph. State and prove Euler's formula for a connected plane graph G with 'v' vertices, 'e' edges and 'f' faces. If all cycles in G have length at least 4, then show that $e \leq 2v-4$, where $v \geq 3$. 1+1+2+1=5

P.T.O.

4. a) Show that if G is a loop free graph and $v \geq 3$, then G has a cycle of even length. 2

   b) Define a tree. Show that if T is a tree in which the neighbour of every leaf has degree at least 3, then some pair of leaves have a common neighbour. 1+2=3

   c) Explain vertex colouring of a graph G. Define chromatic number of G. Show that for any simple graph G

   $$\chi(G) \leq \Delta(G) + 1.$$ 1+1+3=5

5. a) Compute / Count the number of paths of length n in the xy-plane starting from the origin with steps

   $R:(x,y) \to (x+1, y)$,

   $L:(x,y) \to (x-1, y)$ and 4

   $U:(x,y) \to (x, y+1)$.

   It is given that step R is not followed by step L and vice-versa.

   b) State Pigeon-hole principle. Use it to show that if a poset $\langle P_1 \leq \rangle$ has at least n^2+1 elements then it has either a chain of size $n+1$ or an anti-chain of size $n+1$. 1+3=4

   c) Show that if $n+1$ numbers are chosen from $\{1, 2, ..., 2n\}$, then one must divide the other. 2

6. a) State the principle of Inclusion-Exclusion. Use it to count the number of derangements of the permutations of $\{1, 2, ..., n\}$. 1+3=4

   b) State and prove Burnside's theorem on counting. Illustrate the result with an example of your choice. 1+4+1=6

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