B.A./B.Sc. (Honours) Examination 2018

Semester - V

Mathematics

Course: BMC-53

(Probability)

Time: Three Hours | Full Marks: 40

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.
Answer question number 1 and ***any three*** from the rest.

1. Attempt ***any five*** questions. 5×2=10
   a) Show that the probability of occurrence of only one of the events A and B is $P(A)+P(B)-2P(AB)$.
   b) Define Bernoullian and Poisson sequences of trials.
   c) If X is a $\beta_2(\ell,m)$ variate, then show that $Y = 1/X$ is a $\beta_2(m,\ell)$ variate.
   d) Define variance of a random variable X. Does it exist for the Cauchy distribution? Give reasons.
   e) Show that the mode M of the Poisson distribution with mean μ is the integer(s) determined by inequalities: $\mu - 1 \le M \le \mu$.
   f) Prove the Schwartz's inequality for expectations that $\left[E(XY)\right]^2 \le E(X^2)E(Y^2)$.
   g) Define conditional means for a continuous bivariate distribution, and hence define the regression curves for the means.
   h) Define convergence in probability and state the Bernoulli's theorem.

2. a) State the axioms of mathematical probability of an event connected to a random experiment, and give the frequency definition of probability. 2+1
   b) If $\{A_n\}$ is a contracting sequence of events in the event space S, then prove that $P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$. 4
   c) A card is drawn at random from each of two well-shuffled packs of cards. Find the probability of the event that at least one of them is a queen of spades. 3

3. a) State and prove the Bayes' theorem. 1+4
   b) There are three identical urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn 3 white and 3 black balls, and the third urn 5 white and 2 black balls. An urn is chosen at random, and a ball is drawn from it. If the ball drawn is white, what is the probability that the second urn is chosen? 5

4. a) If X is Poisson distributed with parameter μ, then prove that $P(X \le n) = \frac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^n dx$, where n is any positive integer. 3
   b) A point P is taken at random on a line segment AB of length 2a. Find the probability that the area of the rectangle $AP.PB$ will exceed $\frac{1}{2}a^2$. 3

P.T.O.

c) The joint density function of the random variables X, Y is given by $f(x,y) = 2(0 < x < 1,\ 0 < y < x)$. Find the marginal and conditional density functions. Compute $P\left(\frac{1}{4} < X < \frac{3}{4} \Big/ Y = \frac{1}{2}\right)$. 2+2

5. a) Find the mean, median and the mode of a binomial $\left(4, \frac{1}{4}\right)$ variate. 1+1+1

b) Define correlation coefficient $\rho(X,Y)$ of two random variables X and Y.
If the regression lines are $x + 6y = 6$ and $3x + 2y = 10$, find the means and the correlation coefficient. 1+3

c) A continuous distribution has probability density $f(x) = ae^{-ax}$; $0 < x < \infty$ and $a > 0$. Calculate the moment generating function, and hence obtain $E\left(X^k\right)$, where k is a non-negative integer. 3

6. a) State and prove the Tchebycheff's inequality for a continuous random variable X. 1+2

b) In a Poisson sequence of n trials, if f denotes the frequency ratio of successes and $\overline{p} = \frac{1}{n}\sum p_i$, where p_i is the probability of success in the ith trial $(i = 1, 2, \ldots)$, then prove that $f - \overline{p} \xrightarrow[in\ p]{} 0$ as $n \to \infty$. 4

c) State the DeMoivre-Laplace limit theorem. When a sequence of random variables $\{X_n\}$ is said to be asymptotically normal? 2+1

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