B.A./B.Sc. (Honours) Examination 2018
Semester - V
Mathematics
Course: BMC-52
(Differential Equation-II)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer question number **1** and ***any three*** from the rest.

1. Answer ***any five*** questions. 5×2=10

a) The Wronskian of any two solutions of the Ordinary Differential Equation (ODE) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$ is constant. What does the above statement imply about the coefficients P and Q ? Explain.

b) Given $\frac{dy}{dx}=f(x,y),\ y(x=0)=1$, find $f(x,y)$ such that the IVP has many solutions.

c) Show that the nonlinear ODE $y+3x\frac{dy}{dx}+2y\left(\frac{dy}{dx}\right)^3+\left(x^2+2y^2\frac{dy}{dx}\right)\frac{d^2y}{dx^2}=0$ is exact and find its first integral.

d) Define regular singular point of an ODE. Classify the singular point of the ODE $x^2\frac{d^2y}{dx^2}+y\sin x=0.$

e) Interpret the Partial Differential Equation (PDE) $a\,p+b\,q=0,$ (where 'a' and 'b' are constants not both zero) geometrically. $\left[z=z(x,y), p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}\right]$

f) Solve the PDE $4p-3q=0,$ together with the auxiliary condition that $z(0,y)=y^3$.

g) Can you solve $q=p$ in the first quarter of the $xy-$plane with $z(x,0)=f(x)$ and $z(0,y)=g(y)$? Explain.

h) State the order of the following PDEs:
(i) $q-r+1=0,$ (ii) $q-s+t+pz=0$.
$\left[z=z(x,y),\quad r=\frac{\partial^2 z}{\partial x^2},\ s=\frac{\partial^2 z}{\partial x\partial y},\ t=\frac{\partial^2 z}{\partial yz}\right]$
Hence, examine whether they are nonlinear, linear homogeneous or linear inhomogeneous.

2. a) If $y=y_1(x)$ is a solution of the ODE $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$, then show that $y=C_1y_1(x)+C_2y_2(x)$ is the general solution of the given ODE, where

$$y_2(x)=y_1(x)\int\frac{\exp\left[-\int P(x)dx\right]}{\left[y_1(x)\right]^2}dx\,;$$

C_1 and C_2 are two independent arbitrary constants. 5

P.T.O.

b) Use Picard's method of successive approximation to solve the following IVP and compare the result with the exact solution:

$$\frac{dy}{dx}+xy=x,$$
$$y(x=0)=0.$$
3+2

3. a) Use the power series method to solve the ODE $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+2)y=0$ about the ordinary point $x_0=0$. 6

b) Describe a method of finding the general integral of the PDE

$$p\ P(x,y,z)+q\,Q(x,y,z)=R(x,y,z).$$
4

4. a) Determine the unknown function 'f', so that the differential equation

$$(y^2+z^2-x^2)\ dx-2xy\ dy+2xf(z)\ dz=0$$

is integrable and then solve it. 5

b) Obtain the integral surface of the PDE

$$x^2p+y^2q+z^2=0$$

passing through the hyperbola $xy=x+y,\ z=1$. 5

5. a) Solve the simultaneous differential equations:

$$(D+1)x+(D-1)y=e^t,$$
$$(D^2+D+1)x+(D^2-D+1)y=t^2,$$

where $D\equiv\frac{d}{dt}$. 4+1=5

Comment on the nature of the general solution.

b) Show that the following PDEs $xp-yq=x$ and $x^2p+q=xz$ are compatible and hence, find a one – parameter family of common solutions. 2+3=5

6. a) Applying Charpit's method, find the complete integral of the PDE $z^2=pqxy$. 5

b) Solve: $x^2r-y^2t+xp-yq=\log_{2018}x$. 5

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