Use Separate Answer Scripts for each Unit

B.A./B.Sc. (Honours) Examination 2018

Semester - I

Mathematics

Course: BMA-11 (Old) (Allied)

(For Back Candidates)

Time: Four Hours | Full Marks: 60

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks-40)

(Algebra and Trigonometry)

Answer ***any four*** questions.

1. a) State the remainder theorem for polynomials. Find the remainder when $4x^5+3x^3+6x^2+5$ is divided by $2x+1$. 1+2
   b) Solve the equation $2x^3-x^2-18x+9=0$, if two of the roots are equal in magnitude but opposite in sign. 4
   c) If α,β,γ be the roots of the equation $x^3+6x^2+11x-6=0$, find the value of $\sum\alpha^2$, $\sum\alpha^2\beta$. 3

2. a) Find the sum of 99^{th} powers of the roots of the equation $x^7-1=0$. 4
   b) Solve $\exp z=-1$. 4
   c) If n is an integer, then show that $\exp(z+2n\pi i)=\exp z$. 2

3. a) Prove that x^2+x+1 is a factor of $x^{10}+x^5+1$. 3
   b) If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\alpha\beta+\beta\gamma,\ \beta\gamma+\gamma\alpha,\ \gamma\alpha+\alpha\beta$. 4
   c) Find the general values of i^i. 3

4. a) Let A be an m×n matrix. Show that both AA^t and A^tA are symmetric matrices. 2
   b) Solve by matrix method the following system of equations:
   $x+z=0,\ 3x+4y+5z=2,\ 2x+3y+4z=1.$ 4
   c) Three n×n matrices A,B,C are such that $AB=I_n$ and $BC=I_n$. Prove that $A=C$. 2
   d) If A is an invertible matrix, then show that A^t is inventible and $\left(A^t\right)^{-1}=\left(A^{-1}\right)^t$. 2

5. a) Prove that the matrix $\frac{1}{3}\begin{pmatrix}1 & -2 & 2\\ 2 & -1 & -2\\ 2 & 2 & 1\end{pmatrix}$ is orthogonal. Find A^{-1}? 3
   b) If A and B are orthogonal matrices of the same order, then show that AB is orthogonal. 3
   c) Let $\mathbb{Z}$ be the set of all integers. Define a binary operation '•' on $\mathbb{Z}$ by $a\bullet b=a+b+1$. Examine if $(\mathbb{Z},\bullet)$ is a group or not. 4

P.T.O.

6. a) Let $(G,\bullet)$ be a group and $a,b,c \in G$. If $b\bullet a = c\bullet a$, then show that $b = c$. 3

b) In a group $(G,\bullet)$, prove that $(a\bullet b)^{-1} = b^{-1}\bullet a^{-1}$ for all $a,b \in G$. 4

c) Show that $G = \{1, i, -1, -i\}$ forms a group under usual multiplication of complex numbers. 3

Unit-II (Marks-12)

(Two Dimensional Geometry)

Answer ***any two*** questions.

7. Find the angle by which the axes should be rotated so that the equation $ax^2 + 2hxy + by^2 = 0$ becomes another equation in which the term xy is absent. Hence find the angle through which the axes are to be rotated so that the equation $17x^2 + 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1,\ A > 0.$ 5+1

8. If the straight lines joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and $2x + 3y + k = 0$ be at right angles, show that $6k^2 + 5k + 52 = 0$ 6

9. (i) If the chord joining the points $P\left(at_1^2, 2at_1\right)$ and $Q\left(at_2^2, 2at_2\right)$ on the parabola $y^2 = 4ax$ passes through its focus then show that $t_1 t_2 = -1$.

(ii) If (x_1, y_1) and (x_2, y_2) be the ends of a focal chord of the parabola $y^2 = 4ax$ then prove that $y_1 y_2 + 4x_1 x_2 = 0$. 3+3

Unit-III (Marks-08)

(Vector Algebra)

Answer ***any two*** questions.

10. Use vector method to show that the four points $P(1,5,-1)$, $Q(0,4,5)$, $R(-1,5,1)$ and $S(2,4,3)$ are co-planer. 4

11. Show that in a triangle the perpendiculars drawn from the vertices to the opposite sides of it are concurrent. 4

12. Find the scalar area of the parallelogram whose diagonals are the vectors $(3\vec{i} + \vec{j} - 2\vec{k})$ and $(\vec{i} - 3\vec{j} + 4\vec{k})$. 4