B.A./B.Sc. (Honours) Examination 2018

Semester - III

Mathematics

Course: BMC-33 (Old)

(Mechanics-I)

(For Back Candidates)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) A particle moves with a constant acceleration in a straight line. Show that the space-average of the velocity over any distance is $\frac{2}{3}\frac{u_1^2+u_1u_2+u_2^2}{u_1+u_2}$ where u_1 and u_2 are initial and final velocities. 5

   b) A particle is let to fall from rest from a point outside the earth at a distance b from the centre. Prove that the square of the velocity of the particle on reaching the centre is $ga\left(3-\frac{2a}{b}\right)$, where a is the radius of the earth and g is the value of gravity at its surface. 5

2. a) Prove that the composition of two simple harmonic motions having the same period in the same straight line is again a simple harmonic motion. 5

   b) A mass m hangs from a fixed point by a light string and is given a small vertical displacement. If ℓ be the length of the string when the system is in equilibrium and n be the number of oscillations per second, then show that the natural length of the string is $\ell-\frac{g}{4\pi^2n^2}$. 5

3. a) Deduce the expressions for the radial and transverse components of acceleration of a particle moving along a plane curve. 5

   b) A particle describes a path with an acceleration $\frac{\mu}{y^3}$ which is always parallel to the axis of y and directed toward the x-axis. If the particle be projected from a point $(0,a)$ with the velocity $\frac{\sqrt{\mu}}{a}$ parallel to x-axis, then show that the path described by it is a circle. 5

4. a) A particle describes an equiangular spiral $r=ae^{m\theta}$ with constant velocity. Find the components of velocity and of acceleration along the radius vector and perpendicular to it. 5

   b) An insect crawls at a constant rate u along the spoke of a cart wheel, of radius a, the cart moving with a constant velocity v. Find the acceleration along and perpendicular to the spoke. 5

P.T.O.

5. a) Prove that the differential equation of the path of a particle moving in a central orbit under an attractive force F per unit mass is $F = \frac{h^2}{p^3}\frac{dp}{dr}$. 5

   b) If the central orbit be an ellipse under a force towards the centre, then find the law of force. Find also the velocity at any point of the orbit. 5

6. A particle describes a circular path about a centre of force $\left(= \mu / r^n\right)$ at its centre. Discuss the stability of the motion. 5

   b) A particle of mass m is projected vertically under gravity; the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}\left[\lambda - \log_e(1+\lambda)\right]$, where V is the terminal velocity of the particle and λV is the initial velocity. 5

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