Use Separate Answerscripts for each Unit

B.A./B.Sc. (Honours) Examination 2018

Semester – III

Mathematics

Course: BMA-31 (Allied) (Old)

For Back Candidates

Time: Four Hours | Full Marks: 60

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks-24)

(*Improper integrals, Fourier series and Integral transforms*)

Answer ***any two*** questions.

1. a) Show that the integral $\int_0^{\pi/2} \sin x \log \sin x \, dx$ converges and find its value. 4

 b) For what value(s) of n, the integral $\int_a^{\infty} \frac{dx}{x^n}$ is convergent? $(a>0)$. 3

 c) Define Gamma function. Show that $2^n \Gamma\left(n+\frac{1}{2}\right) = 1.3.5...(2n-1)\sqrt{\pi}$ for $n>0$. 2+3

2. a) State the Dirichlet's conditions for the convergence of a Fourier series. 2

 b) Expand the function $f(x) = x + x^2,\ -\pi < x < \pi$, in Fourier series and deduce that

 $$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$$ 3+2

 c) Find the Fourier transform of

 $$f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}.$$ 5

3. a) Find the Laplace transform of $e^{at}\cos bt$ for some constants a and b. Does the Laplace transform of $\frac{\cos at}{t}$ exist? 2+2

 b) Use convolution theorem to evaluate the inverse Laplace transform of

 $$\frac{s^2}{\left(s^2+4\right)^2}.$$ 4

 c) Use the method of Laplace transform to solve the differential equation

 $$\frac{d^2y}{dt^2} + y = t,\ y(0) = 0,\ y'(0) = 1$$ 4

Unit-II (Marks-20)

(*Differential Equations*)

Answer ***any four*** questions.

4. Find the orthogonal trajectories of the family of hypocycloids $x^{2/3} + y^{2/3} = a^{2/3}$, where a is a variable parameter. 5

P.T.O.

5. Solve by the method of variation of parameters the differential equation
$$\frac{dy}{dx}-5y=\sin x.$$ 5

6. Obtain the complete primitive and singular solution of $y=px+\sqrt{1+p^2}$, $p\equiv dy/dx.$ 5

7. Reduce the equation $x^2(y-px)=p^2y$ to Clairaut's form and hence solve the equation $\left(p\equiv\frac{dy}{dx}\right).$ 5

8. Show that the equation $x^3\frac{d^3y}{dx^3}-6x\frac{dy}{dx}+12y=0$ has three linearly independent solution of the form $y=x^r$. 5

9. Solve the differential equation
$$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}-4y=xe^{-2x}.$$ 5

Unit-III (Marks-16)

(*Vector Calculus*)

Answer ***any two*** questions.

10. a) Find the equation of the tangent plane to the sphere $x^2+y^2+z^2=1$ at $(1,0,0)$. 4

b) Let $f(x,y,z)=(x^3+y^3+z^3)xyz$. Find a vector v in $\mathbb{R}^3$ along which direction the absolute value of the rate of change of f is maximum at the point $(1,2,3)$. 4

11. a) Find the divergence of the vector field
$$v(x,y,z)=\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i}+\frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j}+\frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$ 4
at the point $(1,1,1)$.

b) If $f(r)$ is a differentiable function then show that $curl(\vec{r}\,f(r))=\vec{0}$. 4

12. a) Evaluate $\int_{(0,0)}^{(2,1)}(10x^4-2xy^3)dx-3x^2y^2dy$ along the path $x^4-6xy^3=4y^2$, using Green's theorem. 4

b) Verify Stokes' theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary. 4