B.A./B.Sc. (Honours) Examination 2018
Semester - III
Mathematics
Course: BMC-31 (Old)
(Analysis-III)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) If (L_1, U_1) and (L_2, U_2) denote the section of real number α and β respectively, then find the section of $\alpha+\beta$ with proper justifications. 3

   b) When is a set said to be countable? If $f: D \to \mathbb{R}$ be a function and if $D \subseteq \mathbb{R}$ is countable then prove that $f(D)$ is countable in $\mathbb{R}$. 1+2

   c) Prove that the set $\mathbb{R}$ of real numbers is uncountable. 4

2. a) Define a closed set. Prove that union of finite number of closed sets is a closed set. Does the result hold if the number of closed sets is infinite? 4

   b) Find the closure of the following sets with proper justifications:
   (i) $\mathbb{Q}$ (ii) $\mathbb{N}$ (iii) $\mathbb{R}-\mathbb{Z}$ ii) $\mathbb{Q}-\mathbb{Z}$. 4

   c) Let S be a non-empty sub set of $\mathbb{R}$ which is bounded below and $T=\{-x : x \in S\}$. Prove that T is bounded above and $sup\, T = -Inf\, S.$. 2

3. a) When is a set said to be compact? Prove that every compact set in $\mathbb{R}$ is closed and bounded. 5

   b) Give an example of an open cover of $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Does there exist an open subcover of it? Justify your answer. 3

   c) If a set S of $\mathbb{R}$ is bounded below and has no least element, then prove that $Inf\, S \subseteq S'$. 2

4. a) State and prove Euler's theorem for a homogenous function of two variables. 3

   b) If $v=\cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}+\frac{1}{2}\cot v=0$. 3

   c) A function $f:\mathbb{R}^2 \to \mathbb{R}$ is defined by

   $$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} & \text{when } x^2+y^2 \neq 0 \\ 0 & \text{when } x^2+y^2=0 \end{cases}$$ 4

   Evaluate f_{xx}, f_{yx}, f_{xy} and f_{yy} at $(0,0)$.

P.T.O.

5. a) Let $f(x,y)=|xy|^p$, $(x,y)\in\mathbb{R}^2$. Prove that f is differentiable at $(0,0)$ if $p>\frac{1}{2}$. 4

b) Evaluate $\lim_{(x,y)\to(,0,0)} \frac{x^2y^2}{x^2+y^2}$. 3

c) Test the continuity of $f(x,y)$ at $(0,0)$ where

$$f(x,y)=\begin{cases}(x+y)\sin\frac{x}{y} & \text{when } y\neq 0\\ 0 & \text{when } y=0\end{cases}$$ 3

6. a) Define uniform continuity of a real valued function defined on a subset of $\mathbb{R}$. Let $f(x)=x^2$, $x\in\mathbb{R}$, then show that f is uniformly continuous in any closed interval $[a,b]$, but f is not uniformly continuous in $[a,\infty)$. 4

b) Prove that if a function f is continuous over $[a,b]$, then it is bounded and attains its least upper bound and greatest lower bound in $[a,b]$. 4

c) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{8}{x+y+z}$. 2
