Use Separate Answer Scripts for each Group

B.A./B.Sc. (Honours) Examination 2018

Semester - I

Mathematics

Course: BMC-13 (Old)

(Geometry)

(For Back Candidates)

Time: Three Hours Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Gr.-A (Full Marks=16)
(Two Dimensional Geometry)
Answer ***any four*** questions.

1. Find the angle through which the axes are to be rotated so that the equation $7x^2+8xy-5y^2=1$ may be reduced to the form $Ax^2+By^2=1,\ A>0;$ find also A and B. 4

2. Verify whether the equation of the pair of straight lines through the origin perpendicular to the pair of straight lines $ax^2+2hxy+by^2=0$ is $bx^2-2hxy+ay^2=0$. 4

3. Derive the equations of the tangents to the conic $x^2+4xy+3y^2-5x-6y+3=0$ which are parallel to the straight line $x+4y=-2,$ if exists. 4

4. Find the pole of the straight line $2x-5y=4$ with respect to the parabola $y^2=8x$. 4

5. Reduce the equation $6x^2-5xy-4y^2+7x+8y+4=0$ to its canonical form and determine the nature of the conic. 4

6. Show that the straight line $r\cos(\theta-\beta)=p$ in polar coordinate touches the conic $\frac{1}{r}=1+e\cos\theta,$ if $(l\cos\beta-ep)^2+l^2\sin^2\beta=p^2$. 4

Gr.-B (Full Marks=24)
(Three Dimensional Geometry)
Answer ***any three*** questions.

7. Find the angle between two straight lines whose directions cosines are given by $l+m+n=0,\ l^2+m^2-n^2=0,$ if they are not parallel. 4

8. Verify whether the equation $2x^2-y^2+3z^2-xy+7xz+2yz=0$ represents a pair of planes. Find the angle between them, if yes and are not parallel. 4

9. A plane cuts the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (a,b,c). Show that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=k,$ where k is a number to be determined by you. 4

P.T.O.

10. Prove that the plane passing through the point (a,b,c) containing the straight line $x = py + q = rz + s$ can be put into the form

$$\begin{vmatrix} x & py+q & rz+s \\ a & pb+q & rc+s \\ 1 & 1 & 1 \end{vmatrix} = 0.$$ 4

11. Show that by a special choice of coordinate axes the equation of two skew lines can be put in the form $y = \pm m\ x,\ z = c.$ 4

12. Obtain the equation of the sphere through the point $(1,1,2)$ and $(2,-2,3)$ and having its center on the line $2x+3y=0=5x+y-z.$ 4

13. Prove that a general equation of second degree

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$$

may represent a cone provided the condition

$$\begin{vmatrix} a & h & g & u \\ h & b & f & v \\ g & f & c & w \\ u & v & w & d \end{vmatrix} = 0$$ on coefficients a,b,c,f,g,h,u,v,w,d holds. 4

14. Derive the equation of a cylinder whose guiding curve is

$$ax^2+by^2+2hxy+2gx+2fy+c=0,\quad z=0$$

and whose generators are parallel to the line

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}.$$ 4

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