B.A./B.Sc. (Honours) Examination 2018

Semester – I
Mathematics
Course: BMC-12 (Old)
(Algebra-I)
(For Back Candidates)

Time: Three Hours — Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) Without expanding the determinant, prove that
$$\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0.$$ 3

   b) Show that a determinant of a skew-symmetric matrix of odd order is zero. 3

   c) Define an orthogonal matrix. If A be an orthogonal matrix, then show that A is non-singular and $\det A = \pm 1$. 4

2. a) If A be an invertible matrix, then show that A^T is also invertible and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$, where A^T is the transpose of A. 3

   b) Reduce the matrix
$$\begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{bmatrix}$$
to a row-echelon matrix by applying elementary row operations. Hence, find the rank of the matrix. 4

   c) Show that the equations $x + 2y - z = 3,\ 3x - y + 2z = 1,\ 2x - 2y + 3z = 2$ and $x - y + z = -1$ are consistent and hence solve the equations. 3

3. a) Define partial order relation on a non empty set. Show that the relation 'C' (inclusion relation) defined on $P(S)$ the power set of S, is a partial order relation. 4

   b) For any three sets $A, B, C,$ of an universal set S prove that $\left(A \cap B\right) - \left(A \cap C\right) = \left(A \cap B\right) - C.$ 3

   c) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f\left(x\right) = |x|,\ x \in \mathbb{R}$ is neither injective nor surjective. 3

4. a) If $\alpha = \cos\frac{2r\pi}{n} + i\sin\frac{2r\pi}{n}$ and if r and p are prime to n, then show that
$$1 + \alpha^p + \alpha^{2p} + \ldots + \alpha^{(n-1)p} = 0.$$ 3

   b) Express $\cos^6\theta$ in terms of cosine of multiples of θ. 3

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c) If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$, and $x + y + z = 0$, then show that

i) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$,

ii) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3/2$. 2+2

5. a) Prove that the roots of the equation $\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} = x$ are all real. 3

b) If α be a multiple root of the polynomial equation $f(x) = 0$ of order r, then show that α is a multiple root of $f'(x) = 0$ of order $r - 1$. 3

c) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\frac{1-\alpha}{1+\alpha}, \frac{1-\beta}{1+\beta}, \frac{1-\gamma}{1+\gamma}$ and hence find the value of $\frac{(1-\alpha)(1-\beta)(1-\gamma)}{(1+\alpha)(1+\beta)(1+\gamma)}$. 4

6. a) Solve by Cardan's method, $x^3 - 18x - 35 = 0$. 3

b) If $f(x) = x^4 + 6x^3 + 14x^2 + 22x + 5$, find α, β, λ so that $f(x)$ may be expressed in the form $\left(x^2 + 3x + \lambda\right)^2 - (\alpha x + \beta)^2$. Hence solve $f(x) = 0$. 4

c) If a, b, c be three positive real numbers such that the sum of any two is greater than the third, then prove that $abc \geq (a + b - c)(b + c - a)(c + a - b)$. 3

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