# B.A./B.Sc. (Honours) Examination 2018

**Semester - I**

**Mathematics**

**Course: BMC-11 (Old)**

**(Analysis-I)**

(For Back Candidates)

Time: Three Hours | Full Marks: 40

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) If in a certain neighbourhood of $a, f(x) \le g(x) \le h(x)$ and $\lim_{x\to a} f(x) = \ell = \lim_{x\to a} h(x)$, then show that $\lim_{x\to a} g(x)$ exists and is equal to $\ell$. 4

   b) If $f:\mathbb{R}\to\mathbb{R}$ and $f(x) = x - [x]$, where $[x]$ = integral part of $x$ but not greater than $x$, then show that $f$ has a jump discontinuity at $x = 1$ of height 1. 3

   c) State and prove Intermediate Value Theorem for a continuous function $f:[a,b]\to\mathbb{R}$. 3

2. a) If a sequence $\{x_n\}$ converges to $\ell$, then show that the sequence $\{|x_n|\}$ converges to $|\ell|$. 3

   b) Show that every convergent sequence of reals is bounded. Is the converse true? Justify your answer. 4

   c) If the sequence $\{x_n\} \to \ell$ as $n\to\infty$ and $x_n \ge 0 \ \ \forall n$, then show that $\ell \ge 0$. 3

3. a) Show that the sequence $\{x_n\}$ defined by $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is a divergent sequence. 4

   b) Prove that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2x_n}$ for $n \ge 1$ converges to 2. 4

   c) State Bolzano-Weierstrass theorem on sequences. 2

4. a) Show that the supremum and Infimum of a nonempty set $S$ are uniquely determined whenever they exist and $Inf\ S \le Sup\ S$. 3

   b) If $y = x^{2n}$, $n$ is a positive integer, then show that $y_n = 2^n\{1.3.5\ldots(2n-1)\}x^n$. 4

   c) Find the values of $a$ and $b$ such that $\lim_{x\to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3} = \frac{1}{3}$. 3

5. a) If $y = \sin^{-1} x$, then show that

   $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

   Also find $y_n$ at $x = 0$. 5

   b) Evaluate $\lim_{x\to 0}(\sin x)^{2\tan x}$ 3

   c) Write the geometrical interpretation of Rolle's theorem. 2

**P.T.O.**

6. a) State and prove Cauchy's Mean Value Theorem for Differential Calculus.
Deduce Lagrange's Mean Value Theorem from Cauchy's Mean Value Theorem. (1+3)+1

b) If $f$ and $g$ are two differentiable functions in $[a,b]$, then show that, there exists a real number $c$ (where $a<c<b$) such that

$$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a)\begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix}.$$ 3

c) If a function $f$ satisfies the conditions of Lagrange's Mean Value Theorem and $f'(x)=0\ \forall x\in(a,b)$, then show that $f$ is constant in $[a,b]$. 2

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