

B.Sc. (Honours) Examination 2018
Semester – III (CBCS)
Mathematics
Course: SEC-1
(Boolean Algebra and Circuit Design)

Time: Two Hours

Full Marks: 20

Questions are of value as indicated in the margin.
 Notations and symbols have their usual meanings.
 Answer **any four** questions.

1. a) Show that the set of positive divisors of 21 forms a Boolean Algebra with respect to the compositions defined by you.
 Is the result true for the set of positive divisors of 63? 2+1
 - b) In a Boolean Algebra B , prove that for $a, b, c \in B$
 $a \geq b, a \geq c \Rightarrow a \geq b + c$. 1
 - c) Show that in a Boolean Algebra B , $a + a = a \quad \forall a \in B$. 1

2. a) Define Boolean ring.
 Let $(R, +, \cdot)$ be a Boolean ring. If the binary operations \vee, \wedge and the unary operation $'$ are defined by $a \vee b = a + b + ab$, $a \wedge b = ab$, $a' = 1 + a$, 1 being the unit element in R , then show that $(R, \vee, \wedge, 1)$ is a Boolean Algebra. 3
 - b) Let e be an atom and $e \leq x_1 + x_2$, $x_1, x_2 \in B$. Then show that e is a subelement of at least one of x_1 and x_2 . 2

3. a) Show that in a Boolean Algebra B , (i) $1' = 0$ (ii) $a'' = a \quad \forall a \in B$. 2
 - b) In a Boolean Algebra B , show that $a + a.b = a \quad \forall a, b \in B$. 1
 - c) Show that in a finite Boolean Algebra every nonzero element is the sum of all the atoms dominated by it. 2

4. a) Deduce that in a Boolean Algebra B , $a + b = 0 \Rightarrow a = 0$. 1
 - b) In a Boolean Algebra B , prove that $\forall a, b, c \in B$,
 - (i) $a + b = a + c, a' + b = a' + c \Rightarrow b = c$. 1+1
 - (ii) $a + b = a + c, a.b = a.c \Rightarrow b = c$.
 - c) Define atom of a Boolean Algebra.
 If $X = \{1, 2, 3\}$. Find the atoms in the Boolean Algebra $(P(X), \vee, \wedge, 1)$. 1+1

5. a) Let f be a function of three Boolean variables x, y, z defined by $f(x, y, z) = xy + y'$.
 Express $f(x, y, z)$ in disjunctive normal form. 2
 - b) Let a Boolean function f be defined by $f(x, y, z) = xy + yz + zx$. Find the conjunctive normal form of $f(x, y, z)$. 2
 - c) Draw the circuit which realises the Boolean expression $xy + yz + zx$. 1

P.T.O.

6. a) Show that NAND gate is universal gate. 2
b) Draw the following in symbolic diagram 2
$$F = A\bar{B} + AB + BC$$

c) Find $(111)_2 + (111)_2 + (111)_2$, where $(111)_2$ is a binary number. 1
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