M.Sc. Examination, 2018

Semester-I

Physics (Core)

Course Code: MPC-11

(Mathematical Methods-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer ***any four*** *questions.*

1. a) What is an analytic function. Show that the function $2y + ix$ is not differentiable any where in the complex plane. 1+2

b) Show that $\sin(i \ln(\frac{a-ib}{a+ib})) = \frac{2ab}{a^2+b^2}$. 2

c) Locate and identify the type of singularities in the entire complex plane of the functions 2x2.5

i) $[(z-1)(z+i)]^{\frac{1}{2}}$

ii) $[\frac{z^2+4}{z+1}]^{\frac{1}{2}}$.

2 a) Suppose the function $f(z)$ is analytic inside and on a contour C. Obtain the Taylor expansion of the function $f(z)$. 2

b) If $f(z)$ has a pole of order p inside the contour C at the point $z = z_0$ then obtain the series expansion of the function $f(z)$. 3

c) Suppose that $f(z)$ has a pole of order m at the point $z = z_0$. By considering the Laurent series of $f(z)$ about z_0 obtain the general expression for the Residue of $f(z)$ at $z = z_0$. Hence evaluate the residue of the function $\frac{\exp(iz)}{(z^2+1)^2}$ at $z = i$. 5

3 a) Let $f(z)$ be analytic inside and on a simple closed curve C except for the poles at $z = \alpha_1, \alpha_2 ....., \alpha_P$ of order $n_1, n_2 ....., n_P$. If $f(z)$ has zeros at $\beta_1, \beta_2, ..., \beta_N$ of order $m_1, m_2, ..., m_N$ inside the contour C then show that

$$\oint_C dz \frac{\frac{df(z)}{dz}}{f(z)} = \sum_{i=1}^{N} m_i - \sum_{i=1}^{P} n_i.$$ 4

b) Suppose $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C and $|g(z)| < |f(z)|$ inside the contour C. Then show that $f(z) + g(z)$ and $f(z)$ has same number of zeros inside the contour C. 4

c) Show that a polynomial of degree n has exactly n number of zeros. 2

4 a) Evaluate the integral $\frac{1}{2\pi i} \int_0^\infty \sin(x^2) dx$ 4

b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 6

P.T.O.

5.a) Consider a conformal mapping from z plane to the w plane characterized by a transformation $z \rightarrow w = f(z)$, where $f(z)$ is an analytic function. Show that the angle between two intersecting curves in the $z-$ plane equals to the angle between the corresponding curves in the $w - plane$. 4

b) An infinite cylinder of radius a has two halves of its surface (corresponding to semicircular cross sections) maintained at potential V_1 and V_2. Find the electrostatic potential in the interior of the cylinder. 6

6. a) Suppose one of the solution of the differential equation $\frac{d^2y(x)}{dx^2} + a_1(x)\frac{dy(x)}{dx} + a_0(x)y(x) = 0$ is $u_1(x)$. Obtain second solution $u_2(x)$ of the above differential equation. 5

b) Obtain the particular integral of the in-homogeneous differential equation $\frac{d^2y(x)}{dx^2} + a_1(x)\frac{dy(x)}{dx} + a_0(x)y(x) = f(x)$ when the solution of the corresponding homogeneous equations are $u_1(x)$ and $u_2(x)$. 5

7. a) Write down the Legendre equation and obtain the singularities of the Legendre equation. 3

b) Show that $G(x,t) = \frac{1}{\sqrt{1-2x+t^2}}$ can be treated as the generating function for Legendre polynomial. 5

c) Expand the function $\frac{1}{|\vec{r}-\vec{r'}|}$ in terms of Legendre polynomial where, $\vec{r}$ and $\vec{r'}$ are the position vector in the three dimensional space. 2