M.Sc. Semester-IV Examination 2017
Computer Science
Course : MCSO-49
(Quantum Computing)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer Question No. **1** and **any four** from the rest

1. Answer **any eight** from the rest. 1×8=8
   i. State and explain no-cloning theorem. What is the physical implication of it?
   ii. Explain the term Quantum Entanglement, why it is so important in Quantum computing.
   iii. Compare and contrast quantum and classical circuits. Design a circuit that generates Bell's states.
   iv. What is Quantum Fourier Transform? How does it differ from Discrete Fourier Transform?
   v. Compare and Comment on the complexities of Grover Search and classical search algorithms.
   vi. Explain in brief the characteristic features of quantum algorithms in comparison to classical algorithms.
   vii. What are the information available from a qubit?
   viii. What is quantum Oracle?
   ix. What is Hadamard Gate? What are the interesting features of it?

2. i. What is a Unitary Matrix? Give an example. How are the Unitary matrices related to Quantum Gatges? 0.5+0.5+1=2
   ii. X, Y and Z are Pauli matrices. Prove that X, Y and Z are unitary. Find out the products **X.Y, Y.Z, Z.X and X.Y.Z**. 1.5+3+0.5=5
   iii. "Quantum gates are reversible" Explain. 1

3. a) What is a Quantum NOT Gate? What is Control NOT (CNOT) Gate? Give the circuit representation of CNOT gate and explain the role of control bit and target bit.
   b) Explain the function of Toffoli gate (Controlled-Controlled NOT gate) and Exchange Gate with help of matrix representation. Design the corresponding Quantum Circuit.
   c) Name the quantum gates those do not have classical counterparts.

4. i) A state |I> is defined on the basis {|00>, |01>, |10>, |11>}. This state |I> is to be transformed |I'>=

$$\{|00\rangle \rightarrow (|00\rangle + |10\rangle), \quad |01\rangle \rightarrow (|01\rangle + |11\rangle)$$
$$|10\rangle \rightarrow (|00\rangle - |10\rangle), \quad |11\rangle \rightarrow (|01\rangle - |11\rangle)\}$$

   Give the Quantum circuit for such transformation. What is the matrix representation of the required quantum circuit. 2.5+2.5=5
   ii) Show that the matrix represented by the Quantum circuit is unitary. Comment on the nature of the out put states. 2+1=3

P.T.O.

5. a) Give the expression of Discrete Fourier Transformation(DFT) and Quantum Fourier Transformation (QFT). Why the QFT is important for period finding. 1+1+1=3

   b) Consider a 2-qubit $\psi = a_{00}|00> + a_{01}|01> + a_{10}|10> + a_{11}|11>$, write down the Quantum Fourier Transformation of the state. Represent the QFT with a matrix and prove the unitarity. 2+2=4

   c) Give the circuit representation of the QFT. Write down the Quantum circuit for QFT for the 3-qubit states. 0.5.0.5=1

6. Write down clearly the different steps of Shor's algorithm? Apply shor algorithm to factor the number 15. Calculate the complexity of the algorithm. Do yhou think that Shor algorithm is optimal? Comment on it. 2+4+1+1=8

7. (a) Consider a search space consisting of $N=8=2^3$ states. Run Grover's algorithm to search the element X_0 which is represented by the bit string 011. Hence calculate the complexity of such searching. Give the quantum circuit of the problem. 4+1+1=6

   (b) Write down the complexity of Grover's search in general. Compare this with that of classical search. 1+1=2
