B.A. (Honours) Examination, 2018
Semester-IV
Integrated Mathematics and Statistics
Paper: S-1.4.5.P.6 (Subsidiary)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer ***any four*** questions

1. Let $K(t)$ represent the quantity of capital available in an industry at time *t.* Suppose capital depreciates at the rate δ =0.1 and the rate of investment in the industry is a constant $I = 1000$. The capital stock in the industry at time $t = 0$ is 10,000. Find the time path of capital stock in this economy. What is the steady state value of capital in this industry? Does capital converge to this steady state value? 7+1+2

2. Let *y(t)* be the reserve of oil left in an oilfield at time *t*. Suppose extraction reduces reserves at a constant rate equal to α (The rate of decline of reserves is α). If initial reserves at $t = 0$ was 500 million barrels find the expression showing reserves as a function of time. 10

3. Let the production of an economy as a function of the capital be:

$$y = (20 + 0.3k)t^{1/2}$$

Assume that a constant share of output s =0.2 is saved. Capital accumulation in the economy is equal to savings

$$\frac{dk}{dt} = 0.2y$$

Assuming an initial capital stock of $k(0)$=2000, find the solution for capital in this model. 10

4. Suppose stock of a fish species is y quintal in a closed ecosystem. The growth rate of y per month is given by the following equation:

$$\frac{dy}{dt} = y(1 - \frac{y}{11})$$

If 10/11 quintal of fish is harvested from that ecosystem every month, then construct a differential equation of net growth of fish stock per month. Draw a phase diagram of net fish growth rate against fish stock and find the equilibrium values of fish stock for which net growth is zero after such harvesting. Analyse the stability of these equilibrium values. 10

5. (a) Suppose growth rate of population over time is given by $\dot{P} = \frac{t}{t^2+1}P$, where P is the population and t is time. Find the time path of population. From your solution, can you find the exact population number for a given value of t? Explain your answer. 5

(b) For the following differential equation, solve for the time path of y:

$\dot{y} + ty = 4ty^3$ 5

P.T.O.

6. Solve the following linear second-order differential equation, including the constant of integration (using initial conditions $y_0 = 10 \; and \; \dot{y}_0 = 8$)

$$3\ddot{y} + 6\dot{y} - 9y = -18$$ 10

7. Suppose the demand and supply curves in a market is given by

$$q^d = 30 - 2p$$
$$q^s = 10 + \boldsymbol{G}p$$

The price adjustment equation in this model depends not only on excess demand $(q^d - q^s)$ but also on the accumulated inventories from the past since $t = 0$, ie

$$\dot{p} = 0.05(q^d - q^s) - 0.5 \int_0^t [\, q^s(r) - q^d(r)]dr$$

Find the value of G such that price converges to the steady state. 10

8. Solve the linear second-order differential equation

$$\ddot{y} + 6\dot{y} + 9y = 27t$$ 10
