Five Year Integrated M.Sc. Examination, 2017

Semester-II
Course: MT-1-2-1 (Old)
(Mathematics-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Answer ***any four*** questions.

1. a) Prove that $(\vec{A}\times\vec{B}).(\vec{C}\times\vec{D})+(\vec{B}\times\vec{C}).(\vec{A}\times\vec{D})+(\vec{C}\times\vec{A}).(\vec{B}\times\vec{D})=0$. 5

   b) Find the constant 'a' such that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar. 3

   c) Define an irrotational vector. Show that the vector function $\vec{F}=(2x-yz)\hat{i}+(2y-zx)\hat{j}+(2z-xy)\hat{k}$ is Irrotational. 4

   d) Show that $\vec{b}\times\vec{c},\vec{c}\times\vec{a},\vec{a}\times\vec{b}$ are not coplanar if $\vec{a},\vec{b},\vec{c}$. are not coplanar. 3

2. a) Write down the geometrical meaning of gradient of a scalar function. Find the directional derivative of the function $\vec{F}=xe^{xy}+y$ along the direction of $\theta=\dfrac{2\pi}{3}$. 5

   b) If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\vec{i}+2\vec{j}+\vec{k}$ and $\vec{c}=3\hat{i}+\hat{j}$, find "t" such that $\vec{a}+t\vec{b}$ is perpendicular to $\vec{c}$. 3

   c) Find the projection of the vector $4\hat{i}-3\hat{j}+\hat{k}$ on the line passing through the points (2, 3, -1) and (-2, -4, 3). 4

   d) Sketch the vector field on R^2 for $\vec{F}(x,y)=-y\hat{i}+x\hat{j}$. 3

3. a) Show that $\left(2\hat{i}-2\hat{j}+\hat{k}\right)/3$, $\left(\hat{i}+2\hat{j}+2\hat{k}\right)/3$ and $\left(2\hat{i}+\hat{j}-2\hat{k}\right)/3$ are mutually orthogonal unit vectors. 5

   b) Find a potential function for the vector field, $\vec{F}=2xy^3z^4\hat{i}+3x^2y^2z^4\hat{j}+4x^2y^3z^3\hat{k}$. 5

   c) Find an equation for the tangent plane to the surface $xz^2+x^2y=z-1$ at the point (1,-3, 2). 5

4. a) Determine the order and degree of $\left(1+\left(\dfrac{dy}{dx}\right)^4\right)^{\frac{1}{3}}=\dfrac{d^2y}{dx^2}$. 3

   b) Form the differential equation of the family of curves $\dfrac{x^2}{a^2+\lambda}+\dfrac{y^2}{b^2+\lambda}=1$ for different values of λ. Determine whether the differential equation is linear or nonlinear. 5+1

   c) Define linear and nonlinear differential equations. Explain with example that every linear differential equations is of first degree, but the converse is not true. 6

5. a) Show that the equation $(x^3-3x^2y+2xy^2)dx-(x^3-2x^2y+y^3)dy=0$ is exact and find the solution if $y=1$ when $x=1$. 6

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b) Solve: $x\frac{dy}{dx}+y=y^2\log x$. 4

c) Find the integrating factor of $xdy-ydx=0$. 2

d) Show that $y=\cos x$ is a solution of $y\frac{d^2y}{dx^2}-\sin x\frac{dy}{dx}+2\cos x\, y=1$ 3

6. a) Reduce the differential equation $\left(x\frac{dy}{dx}-y\right)\left(x-y\frac{dy}{dx}\right)=2\frac{dy}{dx}$ to Clairaut's form by the substitution $x^2=u, y^2=v$ and find its solution. 5

b) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$. 5

c) Solve: $\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+8\frac{d^2y}{dx^2}-8\frac{dy}{dx}+4y=0$ 5