

**B.A. (Honours) Examination, 2018**

**Semester-V**

**Economics**

**Course: H-12 (Core)**

**(Mathematical Economics II)**

**Time: Three Hours**

**Full Marks: 40**

Questions are of value as indicated in the margin

Answer *any four* questions.

1.(a) The demand and supply equations of a commodity are given as  $P = 200 - 0.02Q_d^2$  and  $P = 100 + 0.02Q_s^2$  respectively. Equilibrium is attained by equating demand and supply. Find the equilibrium price and the total Consumer Surplus. 5

(b) Suppose a plantation crop survives for two years from the time of plantation. The initial cost for planting the trees is Rs. 2000 per Bigha. Crops can be harvested once after the completion of first year and again once after the completion of second year. After the second harvest, the plants die. The estimated net revenue per Bigha (after deducting harvesting and marketing costs) for these two harvests are Rs. 1100 and Rs. 1210 respectively. If the market rate of interest is 10%, find the present value of profit per Bigha for this crop. 5

2. Consider a commodity where both the demand and supply quantities are functions of its current price:

$$Q_{dt} = \alpha - \beta P_t \quad (\alpha, \beta > 0)$$

$$Q_{st} = -\gamma + \delta P_t \quad (\gamma, \delta > 0)$$

However, the price adjustment is made by suppliers and in each period the price is set at a discount over the previous period's price depending on the stock (inventory) of the commodity accumulated from the previous period. The price setting mechanism is given by the following equation:

$$P_{t+1} = P_t - \sigma (Q_{st} - Q_{dt}) \quad (\sigma > 0)$$

(i) Find out the time path of price and the equilibrium price.

(ii) Find the nature of time path of price if  $0 < \sigma < 1 / (\beta + \delta)$  and show the time path graphically (assume initial price to be above the equilibrium price) 6 + 4 = 10

3. Show that business cycles can occur endogenously in a closed market economy with the help of an appropriate macro model using second order difference equations. 10

4. (a) A person needs a daily diet that contains at least 300 grams of carbohydrates, 75 grams of fat and 150 grams of protein. Two foods A and B are available. Food A costs Rs. 20 per kg and food B costs Rs. 40 per kg. In 100 grams of food A, there are 80 grams of carbohydrate, 15 grams of fat and 5 grams of protein. In 100 grams of food B, there are 35 grams of carbohydrate, 10 grams of fat and 55 grams of protein. Formulate the persons' food choice problem mathematically as an LPP. 4

**P.T.O.**

(2)

(b) What is a basic feasible solution (BFS) in the context of an LPP? Rewrite the constraints of the following LPP after introducing appropriate slack variables. How many BFS are there for this problem?

$$\begin{aligned} &\text{Maximize } Z = 4x_1 + 7x_2 \\ &\text{Subject to, } \quad 2x_1 + x_2 \leq 100 \\ &\quad \quad \quad 10x_1 + 10x_2 \leq 800 \\ &\quad \quad \quad 2x_1 + 4x_2 \leq 200 \quad \quad x_1 \geq 0 ; x_2 \geq 0 \end{aligned} \quad \quad \quad 2+2+2=6$$

5. (a) Find the optimum solution of the LPP given in the previous question. If the right hand side of the constraints stands for resources, which one of them will not be fully utilized in the optimum? 5+1=6

(b) Find the dual of the following LPP:

$$\begin{aligned} &\text{Maximize } Z = 2x_1 + x_2 + 3x_3 \\ &\text{Subject to } \quad x_1 + x_2 + 2x_3 \leq 5 \\ &\quad \quad \quad 2x_1 + 3x_2 + 4x_3 = 12 \\ &\quad \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned} \quad \quad \quad 4$$

6. Construct a maximization LPP with three variables and two constraints in standard form. State and prove the theorem of complementary slackness in this set up. 10

7. (a) Suppose two commodities are produced with the following fixed coefficient production functions:

$$X_1 = \text{Min} \left[ \frac{X_{11}}{a_{11}}, \frac{X_{21}}{a_{21}} \right] \quad \text{and} \quad X_2 = \text{Min} \left[ \frac{X_{12}}{a_{12}}, \frac{X_{22}}{a_{22}} \right]$$

Where,

$X_{ij}$  = amount of i-th commodity that flows into the production of j-th commodity as input  
 $a_{ij}$  = requirement of i-th commodity to produce one unit of j-th commodity (technological coefficients)

Draw the isoquant map for each commodity and show the expansion paths. Indicate the slopes of the expansion paths in the figures. 2+2=4

(b) The following is the technology matrix for a two-sector open Input-Output model:

$$\begin{pmatrix} 0.4 & 0.2 \\ 0.3 & 0.1 \end{pmatrix}$$

Check whether the Hawkins-Simon condition is satisfied or not. What is the optimum gross output level of the two commodities if the final demands are 40 and 60 units respectively?

2+4=6

8. Under standard assumptions of a two-sector open Input-Output model, show how you can obtain the consumption possibility frontier with a given quantity of exogenous labour supply. 10