

B.A. (Honours) Examination, 2018
Semester-III
Integrated Mathematics and Statistics
Paper: S-1.3.5.P.5 (Subsidiary) (Old Syllabus)

(For Back Candidates)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Answer *any four* questions.

1. (a) Find the limits of the following functions. If limits do not exist find the left hand and right hand side limits.

(i) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{2x-1}-1}$ (ii) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}$ (iii) $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)$

- (b) Find the point of discontinuity (if any) of the following functions.

$f(x) = \tan x, g(x) = \frac{x-2}{x^2-4}$ 6+4=10

2. (a) State and prove Rolle's theorem

(b) Find the derivative from the first principles $f(x) = \log x$ 5+5=10

3. (a) Calculate the derivatives

(i) $\frac{1-\tan x}{1+\tan x}$ (ii) $\sin \cdot \log \sqrt{x}$

- (b) Find the tangent line at $x = 1$ of $f(x) = \frac{x}{x-2}$

- (c) Let $y = u(x)v(x)$

Find y', y'' and y''' 4+3+3=10

4. (a) Expand the function $f(x+h)$ around $f(x)$ by Taylor's series upto the first three terms. Then explain why the second derivative of a concave function is negative.

6+4=10

- (b) Does the function $f(x) = 2x - \log x$ have a maximum and or a minimum? If so at what values of x . 6+4=10

5. (a) State and prove the fundamental theorem of (integral) calculus.

(b) Directly estimate $\int_3^6 x^2 dx$ as the area under the curve $f(x) = x^2$ between $x=3$ and $x=6$. 5+5=10

6. (a) Evaluate the following Integrals.

(i) $\int 10e^x dx$ (ii) $\int \left(x^{1/2} + 5x^{-2/3} \right) dx$. 3+3+4=10

- (b) Evaluate the following integrals and use the information provided to determine the constant of integration

(i) $F(x) = \int 2x dx, F(3) = 0$ (ii) $F(x) = \int x^{1/2} dx, F(0) = 5$

- (c) Evaluate the definite integral

(i) $\int_1^2 6xe^{x^2} dx$ (ii) $\int_0^{64} \left(x^{1/2} + 5x^{-2/3} \right) dx$

P.T.O.

(2)

7. (a) Use the substitution rule to evaluate the following integral

(i) $\frac{6x^2+8}{(x^3+4x)^2} dx$ (ii) $\int (e^{4x^3} + x^2)(12x^2 e^{4x^3} + 2x) dx$ 5+5=10

(b) Use the technique of integration by parts to evaluate the following integrals

(i) $\int \frac{x^3}{\sqrt{1+x^2}} dx$ (ii) $\int x^3 e^x dx$ 5+5=10

8. Consider the function of several variables

$$f(x_1, x_2) = x_1^{1/2} x_2^{1/3} + x_1^{1/3}$$

(a) Is this function homogenous? If so, of what degree.

(b) Is this function convex or concave?

(c) Find the total derivative of this function. 4+3+3=10
