**B.Sc. (Honours) Examination, 2018**
**Semester-IV**
**Statistics**
**Course : BSC-41**
**(Estimation Theory)**

**Time : 3 Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) Let $X_1,\ldots,X_n$ be i.i.d. Bernoulli(p) random variables and let $S_n = \sum_{i=1}^{n} X_i$

(i) Show $\frac{S_n(S_n - 1)}{n(n-1)}$ is an unbiased for $p^2$.

(ii) Show that $\frac{S_n(n - Sn)}{n(n-1)}$ is an unbiased estimator of pq and has a smaller variance than the estimator $\frac{Sn}{n}\left(1 - \frac{Sn}{n}\right)$. 2+4=6

(b) Let $X_1,\ldots,X_n$ be i.i.d. Poisson ($\lambda$) variables.

(i) Show that $t_n = \bar{X}_n^2 - \bar{X}_n$ is a biased estimator of $\lambda^2$.

(ii) Also, show that there is no exactly unbiased estimator of $1/\lambda$.

2. (a) "Consistent estimator is unbiased" – is the statement true? Justify your answer with suitable example.

(b) Let t be a consistent estimator for $\theta$ and let $\psi(\theta)$ be a continuous function of $\theta$. Prove that $\psi(t)$ is a consistent estimator of $\psi(\theta)$.

(c) Let $(X_1, X_2,\ldots,X_n)$ be a random sample from a rectangular distribution $f(x_i\theta) = \frac{1}{\theta}$, $0 < x < \theta$. Show that the geometric mean.

$$t(X) = \left(\prod_{i=1}^{n} Xi\right)^{1/n}$$ is a consistent estimator of $\theta e^{-1}$. 2+3+5=10

3. (a) Let $X_i$ (I = 1, 2, …,n) be a random sample from the lognormal distribution

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(\log x - \mu)^2\right\}$$

Find a sufficient statistic for (i) μ when σ is known.

(ii) σ when μ is known (iii) (μ,σ)both unknown

P.T.O.

(b) Let X be a random variable with p.d.f.

$f(x;\theta)=\frac{1}{2};\theta<x<\theta+2$, where θ is any real number. Does there exist any sufficient statistic for θ ? Reason your answer. 6+4=10

4. (a) What do you mean by most efficient estimator?

(b) If t is a most efficient estimator and $t'$ a less efficient estimator with efficiency e and the correlation coefficient between $t$ and $t'$ is, $\rho$ show, by considering the estimator $t''$ defined by $(1+e-2\rho\sqrt{e})t''=(1-\rho\sqrt{e})t+(e-\rho\sqrt{e})t'$, that $\rho=\sqrt{e}$.

(c) Consider a sample of size 1 from Poisson ($\lambda$). Suggest an unbiased estimator of $e^{-\lambda}$. Deduce Camar-Rao lower bound of $\psi(\lambda)=e^{-\lambda}$. 2+4+4=10

5. Let $X_1, X_2,\ldots,X_n$ be i.i.d. $G\left(1,\frac{1}{\alpha}\right)$.

(i) Prove that the estimator $\frac{(n-1)}{n\bar{X}}$ is the UMVUE for α with variance $\frac{\alpha^2}{n-2}$.

(ii) Show that the Cammer-Rao bound for an unbiased estimator of α is $\frac{\alpha^2}{2}$. 5+5=10

6. (a) Let $X_1, X_2,\ldots,X_n$ be a sample from $U\left[\theta-\frac{1}{2},\theta+\frac{1}{2}\right]$. Find the MLE of $\theta$ and comment

(b) For a bivariate normal $BN(O,O,\sigma_1^2,\sigma_2^2,\rho)$, deduce the MLE for $\sigma_1^2,\sigma_2^2$ and $\rho$ on the basis of n pairs of sample $(X_1,Y_1), (X_2,Y_2),\ldots,(X_n,Y_n)$. 3+7=10

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