

Undergraduate Examination, 2018
Semester-II
Statistics
Generic Elective Course (GEC-2)
(Introductory Probability)

Time : 3 Hours

Full Marks : 40

Questions are of value as indicated in the margin

Question No.1 is compulsory. Answer **any three** from the rest.

1. Choose the correct alternative/alternatives (**any five**) : 5×2=10

(a) Let X be a random variable with the moment generating function

$$M_X(t) = \frac{1}{216}(5 + e^t)^3, t \in \mathbb{R}$$

Then $P(X > 1)$ equals

(i) $\frac{2}{27}$ (ii) $\frac{1}{27}$ (iii) $\frac{1}{12}$ (iv) $\frac{2}{9}$

(b) Let X be a discrete random variable with the p.m.f.

$f(x) = k(1 + |x|)^2$, $x = -2, -1, 0, 1, 2$; where k is a real constant. Then $P(X = 0)$ equals

(i) $\frac{1}{9}$ (ii) $\frac{2}{27}$ (iii) $\frac{1}{27}$ (iv) $\frac{1}{81}$

(c) Suppose A & B are events with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.2$. Then

$P(B^c | A \cup B)$ is equal to

(i) $\frac{1}{2}$ (ii) $\frac{1}{27}$ (iii) $\frac{1}{12}$ (iv) $\frac{2}{9}$

(d) Let E & F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$ and $P(E|F) > P(E)$. Which of the following statements is (are) true?

(i) $P(F|E) > P(F)$ (ii) $P(E|F^c) > P(E)$ (iii) $P(F|E^c) < P(F)$ (iv) E & F are independent.

(e) Let the random variable $X \sim \text{Bin}(5, p)$ such that $P(X=2) = 2 P(X=3)$. Then the variance of X is

(i) $\frac{10}{3}$ (ii) $\frac{10}{9}$ (iii) $\frac{5}{3}$ (iv) $\frac{5}{9}$

(f) The chance that Dr. A will diagnose a disease X correctly is 0.6. The chance that a patient will die by his treatment after correct diagnosis is 0.4 and the chance of death by wrong diagnosis is 0.7. A patient of Dr. A, who had disease X, died. The probability that his disease was diagnosed correctly is

(i) $\frac{6}{25}$ (ii) $\frac{7}{25}$ (iii) $\frac{6}{7}$ (iv) $\frac{6}{13}$

P.T.O.

(2)

2. (a) Distinguish between pairwise independence and mutual independence of events. Show that the former doesn't imply the later by an example. 2+3=5
- (b) A fair die is thrown n times. Find the probability that the numbers 1,2...6 appears at least once. 5
3. (a) Two persons, A and B respectively toss (n+1) & n coins independently. Find the probability that
- (i) they will have equal number of heads.
- (ii) A will have more heads than B. 6
- (b) Let X be a random variable with the distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8}, & 0 \leq x < 2 \\ \frac{x^2}{16}, & 2 \leq x < 4 \\ 1, & x > 4 \end{cases} \quad 4$$

Find E(X).

4. (a) Player P₁ tosses 4 fair coins and P₂ throws a fair die independently of P₁. Find the probability that the number of heads observed is more than the number on the upper face of the die. 4
- (b) Let X be a random variable with p.d.f. $f(x) = 6x(1-x)$; $0 < x < 1$ 0, otherwise
- Find a lower bound of $\rho\left(\left|X - \frac{1}{2}\right| \leq \frac{1}{2}\right)$ using chebyshev's inequality. 6
5. (a) Find the general expression for the even order central moments of normal distribution with parameters μ and σ . 5
- (b) Prove that $1 - \Phi(x) \leq \frac{1}{x} \phi(x)$, symbols have their usual meanings. 5
6. Suppose the joint p.d.f of X & Y is given by

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & \text{if } 0 < x < 1, \quad 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the marginal distributions of X & Y. 4
- (b) Find the conditional distributions of Y given X = x and of X given Y = y 4
- (c) Also give the regression equation of Y on X. 2
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