B.Sc. (Honours) Examination, 2018
Semester-II
Statistics
Course : CC-4
(Algebra)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Question No.1 is compulsory and **any five** from the rest.

1. Answer **any five** of the following : 2×5=10

(a) Write down the roots of the equation $x^7 - 1 = 0$.

(b) Prove that $|\underset{\sim}{a} - \underset{\sim}{b}| > 0$, unless $\underset{\sim}{a} = \underset{\sim}{b}$.

(c) Show that the diagonal elements of a Hermitian matrix are always real.

(d) Show that every nilpotent matrix is singular.

(e) Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

(f) A = $\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$, then express A^{-1} in the form $\alpha I + \beta A$. Where α, β are constants and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(g) What do you mean by a subspace of a vector space?

(h) For a diagonal matrix A, log (det A) = tr (log A), check the validity of this statement. Here logA denotes the A matrix with logarithm diagonal elements.

2. (a) Reduce the reciprocal equation $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$ to a reciprocal equation of the standard form and solve it. 5

(b) If α, β, γ are the roots of cubic equation $x^3 + px^2 + qx + r = 0$ then print the value of

(i) $\Sigma\alpha^3$ (ii) $\Sigma\alpha^4$ (iii) $\Sigma\frac{1}{\alpha^2}$. 5

3. (a) Solve the cubic equation $x^3 - 15x^2 - 33x + 847 = 0$. 5

(b) Define basis of a vector space. Show that the representation of any vector in terms of the vectors in a given basis is unique. 5

4. (a) Write short notes on elementary matrices. 5

(b) If A and B are square matrices such that AB = A and BA = B. Then show that A', B' are idempotent. 5

P.T.O.

5. (a) If A be a square matrix of order n, then show that $aij(adyjA) = (\det A)^{n-2}$

(b) If A and B be invertible matrices of the same order then show that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$. 5

6. (a) Show that $\begin{vmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ b & b & a & \dots & b \\ .. & .. & .. & & \\ b & b & b & .. & a \end{vmatrix}_{p\times p} = [a+(p-1)b](a-b)^{p-1}$ 5

(b) Show that $\begin{vmatrix} a^4 & a^2 & a & 1 \\ b^4 & b^2 & b & 1 \\ c^4 & c^2 & c & 1 \\ d^4 & d^2 & d & 1 \end{vmatrix} = (a+b+c+d)\begin{vmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{vmatrix}$ 5

7. (a) Determine the conditions for which the system of equations has
(i) only one solution (ii) no solution (iii) many solutions. 5

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x + ay + 3z = b+1$$

(b) A linear mapping $T : \mathbb{R}^3 - \mathbb{R}^4$ is defined by
$T(x_1, x_2, x_3) = (x_2 + x_3, x_3 + x_1, x_1 + x_2, x_1 + x_2 + x_3), (x_1, x_2, x_3) \in \mathbb{R}^3$. Find Ker T. Verify that the set
{T(1, 0, 0), T (0, 1, 0), T (0, 0, 1)} is linearly independent in R^4.

8. (a) State and Prove Cayley-Hamilton theorem. 6

(b) Find a g-inverse of the matrix A = $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 0 & -2 & 4 \\ -1 & 1 & 1 & 3 \\ -2 & 2 & 2 & 6 \end{bmatrix}$ 4

9. (a) Show that $\begin{bmatrix} I_m & O_{m\times n} \\ A_{n\times m} & I_n \end{bmatrix}^{-1} = \begin{bmatrix} I_m & O_{m\times n} \\ -A_{n\times m} & I_n \end{bmatrix}$ 5

(b) If a matrix A be you equivalent to a row echelon matrix having *r* non-zero rows, then show that rank(A) = *r*. 5

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