B.Sc. (Honours) Examination, 2018
Semester-II
Statistics
Course : CC-3

(Probability and Probability Distribution)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Notations have their usual meanings. Answer **any four** questions

1. (a) Let $\Omega = \{1,2,3,4\}$. Write down the sigma-field of subsets of Ω. Let(Ω, $\mathcal{A}$, P) be a probability space and A, B, C $\in \mathcal{A}$. Suppose that P(A) = 0.6, P(B) = 0.5, P(C) = 0.4, $P(A \cap B) = 0.3, P(A \cap C) = 0.2, P(B \cap C) = 0.2$ and $P(A \cap B \cap C) = 0.1$. Find

   $P(A \cup B \cup C)$ and $P(A^c \cap B^c \cap C^c)$. 2+3=5

   (b) For independent events $A_1, A_2, \ldots, A_n$, Show that

   $$P(\cap_{i=1}^{n} A_i) \leq e^{-\Sigma_{i=1}^{n} P(A_i)}$$ 5

2. (a) Let the random variables (X_1, X_2) be distributed according to the following p.mf.

   $$f(x,x) = \begin{cases} \dfrac{x_1 + 2x_2}{18}, & if\ (x,x) \in \{1,2\} \times \{1,2\} \\ 0, & Otherwise \end{cases}$$

   Determine the conditional mean and conditional variance of X_2 given

   $X_1 = x_1, x_1 \in \{1,2\}$.

   (b) The joint p.d.f of (X, Y) is given by

   $$f_{XY}(x,y) = \begin{cases} ce^{-(2x+3y),} & if\ 0 < x < y < \infty \\ 0, & otherwise, \end{cases}$$

   Where c is a real constant.

   (i)Determine the constant c,

   (ii) Find the marginal pdfs of X and Y,

   (iii) Verify whether X and Y are independent or not. 4+6=10

3. (a) Let X be a random variable following normal distribution with mean μ and variance σ^2. Then show that

   $$\mu_r = \begin{cases} 0 & if\ r\ is\ odd \\ (r-1)(r-3)...3.1.\sigma^r & if\ r\ is\ even \end{cases}$$

   (b) If X and Y are independent normal variates with means 8.5 and variances 16.9 respectively. Determine λ such that $P(2X + Y \leq \lambda) = P(4X - 4Y \geq \lambda)$. 5+5=10

p.t.o.

4. (a) Show that Poisson distribution is limiting form of the binomial distribution when number of Bernoulli trials is large and probability of success is very small.

   (b) Find moments of the random variable that has the moment generating function

   $M(t) = (1-t)^{-3}, t < 1$. 5+5=10

5. (a) Three coins C_1, C_2 and C_3 have probabilities of coming up of a head as1/2, 1/3 and ¼ respectively. A player chooses one of the three coins in such a way that the probability of choosing coin C_1 is 1/3 and that of choosing coins C_2 and C_3 are1/4, and 5/12 respectively.

   He flips the chosen coins 4 times. Find the probability of getting

   (i)no head, (ii) at least one head and (iii) exactly three heads.

   (b) Consider a sample of size 3 drawn with replacement from an urn containing 3 white, 2 black and 3 red balls. Let the random variables X_1 and X_2 denote the number of white balls and the number of black balls respecting in the sample. Determine whether or not X_1 and X_2 are independent. 5+5=10

6. (a) State and prove properties of a distribution function.

   (b) Consider four coding machines M_1, M_2, M_3 and M_4 producing binary codes 0 and 1. The machine M_1 produces code 0 and 1 with respective probabilities 1/4 and 3/4. The code produced by machine M_K is fed into machine M_{K+1} (K=1,2,3) which may either leave the received code unchanged or may change it. Suppose that each of the machines M_2, M_3 and M_4 change the received code with probability that the machine M_4 has produced code 1, find the conditional probability that the machine M_1 produced code 0. 6+4=10

7. (a) If *m* things are distributed among '*a*' men and '*b*' women, find the probability that the number of things received by men is odd.

   (b) A man with *n* keys wants to open his door and tries independently and at random. Find the expectation and variance of the number of trials he will need if unsuccessful keys are eliminated from further selection. Assume that only one key fits the door. 5+5=10

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