

B.Sc. (Honours) Examination, 2018
Semester-II
Statistics
Course : BSC-22 (Old)
(Real Analysis and Numerical Analysis)

Time : 3 Hours

Full Marks : 40

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) State and prove Cauchy's first limit theorem. Discuss if the converse of the theorem is true or not. 5+2
 (b) Using Cauchy's first limit theorem, show that if $\{x_n\} \rightarrow l$, then 3
 $(x_1, x_2, \dots, x_n)^{1/n} \rightarrow l$ as $n \rightarrow \infty$, where $x_1, \dots, x_n > 0$
2. (a) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$, is conditionally convergent. Examine the convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$, $x > 0$. 3+4=7
 (b) If $\{x_n\} \rightarrow l, \{y_n\} \rightarrow m$, then show that $\{x_n^2 + y_n\} \rightarrow l^2 + m$. 3
3. Discuss the convergence of the following series.
 (a) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$ (b) $\left(\frac{1}{2}\right)^2 + \left(\frac{1.3}{2.4}\right)^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 + \dots$
4. (a) Define limit superior and limit inferior of a sequence of real numbers. 2
 (b) Show that a bounded sequence $\{x_n\}$ is converge if and only if $\overline{\lim} x_n = \underline{\lim} x_n$. 4
 (c) Suppose $x_n = \left(1 - \frac{1}{n^2}\right) \sin \frac{n\pi}{3}$, find two subsequences of $\{x_n\}$ such that one of which converges to the upper limit and other converges to the lower limit. 4
5. (a) Describe Newton's backward interpolation formula. 5
 (b) Using the method of separation of symbols (or, otherwise) show that 5

$$u_0 - u_1 + u_2 - u_3 + \dots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_3 + \dots$$
 Here $u_n = Eu_{n-1}$
6. (a) Find $\Delta^n(e^{ax}), \frac{\Delta}{E}(x^2)$. 5
 (b) Show that $[x_0, x_1, \dots, x_n] = \frac{\Delta^n y_0}{n!h^n}$. (Notations have their usual meaning) 5

(2)

7. (a) Using Euler Maclourin's summation formula, obtain the sum of the expression

$$\sum_{i=1}^n i^p .$$

5

(b) Describe the method of iteration and its convergence criterion.

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