B.A. (Honours) Examination, 2018

Semester-III

Integrated Mathematics and Statistics

Paper: S-1.3.5.P.4 (Subsidiary) (Old Syllabus)

(For Back Candidates)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Unit-I

Answer ***any two*** from the following

1. (a) Briefly explain the concept of vector addition and scalar multiplication with diagram.

   (b) Use vector notation to prove that the diagonal of a rhombus are orthogonal to each other.

   (c) Draw a plane and show the path you would traverse were you to start at (–1, –3) then displace yourself first by vector (1, –3) and then by vector (–1, –3). 4+3+3=10

2. (a) Use vector method to prove that the medians of a triangle are concurrent.

   (b) For any two vectors u and v in R^n

   Prove that $\|u + v\| \leq \|u\| + \|v\|$ for all u, v. 5+5=10

3. (a) For a rectangular 2' ×3' ×4' box find the angle that the longest diagonal makes with the 4' side.

   (b) Define Cross product. Use the Cross product to find a vector perpendicular to both u and v where, (i) $u = (1, 0, 1)$, $v = (1, 1, 1)$ (ii) $u = (1, -1, 2)$, $v = (0, 5, -3)$

   4+6=10

4. (a) Two points X (a_1, b_1, c_1) and Y (a_2, b_2, c_2) in R3 are given. How do you parameterize the line segment joining these two points?

   (b) Transform the following parametric form into the form $x_2 = mx_1+b$

   $x_1 = 3 + t$

   $x_2 = 5$

   (c) Write the non-parametric equation for the plane given by

   $x = 1+s+t$, $y = 2+3s+4t$, $z = s-t$ 4+2+4=10

Unit-II

Answer ***any two*** from the following

1. (a) Define a non-singular Matrix.

   (b) Solve by Cramer's Rule

   $x+y+z = 4$

   $2x-y+3z = 14$

   $x-2y+z = 7$

P.T.O.

(c) Find the inverse of $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ 2+5+3=10

2. (a) Find an upper triangular Matrix A such that $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$

(b) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ Find a 2×3 matrix B with distinct non-zero entries such that AB=0

(c) Let $A = \begin{bmatrix} 5 & 2 \\ 0 & K \end{bmatrix}$ Find the value of K for which A satisfies $A^2 - 7A + 10I = 0$.

3+4+3=10

3. (a) Prove without expanding

$$\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)^3$$

(b) Show that $\begin{bmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{bmatrix}$ is a perfect square 5+5=10

4. (a) State Cayley-Hamilton Theorem

(b) Use Cayley-Hamilton theorem to find A^{-1}

Where $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$

(c) Find the eigen values and eigen vectors of the Matrix $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ 2+4+4=10
