

B.A. (Honours) Examination, 2018
Semester-I (CBCS)
Economics
Course: CC-2 (Core)
(Mathematical Methods for Economics-I)

Time: Three Hours

Full Marks: 60

Questions are of value as indicated in the margin
Answer **any four** questions taking two from each Unit.

Unit-I

1. (a) Define $Lt f(x)$.
 $x \rightarrow a$
- (b) Show that $Lt_{x \rightarrow 0} \frac{1}{x}$ does not exist. Give a geometrical point of view.
- (c) A function $\phi(x)$ is defined as follows
 $\phi(x) = x^2$ when $x < 1$
 $= 2.5$ when $x = 1$
 $= x^2 + 2$ when $x > 1$
Does $Lt_{x \rightarrow 1} \phi(x)$ exist?
- (d) Given $f(x) = ax^2 + bx + c$ show that
 $Lt_{x \rightarrow 0} \left[\frac{f(x+b) - f(x)}{b} \right] = 2ax + b$
- (e) State Cauchy's necessary and sufficient condition for the existence of a limit. Using Cauchy's Theorem to prove that $Lt_{x \rightarrow 0} \frac{1}{2+e^{1/x}}$ does not exist. 2+3+3+3+4=15
2. (a) Define continuity of a function.
- (b) Test the continuity of $f(x) = \frac{1}{5+e^{\frac{1}{x-2}}}$ at $x = 2$.
- (c) Show that the function $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$.
- (d) $Lt f(x)$ be defined in $1 \leq x < \infty$
 $f(x) = \begin{cases} 2 - x & \text{when } 1 \leq x \leq 2 \\ 3x - x^2 & \text{when } x > 2 \end{cases}$
Is the function derivable at $x = 1$? at $x = 2$?
- (e) $f(x) = x + 1, 0 \leq x \leq 1$
 $= 3 - x \quad 0 \leq x \leq 2$
Show that $f(x)$ is not derivable at $x = 1$ 2+4+3+4+2=15
3. (a) Find the derivatives using the first principle (i) $f(x) = e^{\sqrt{x}}$ (ii) $f(x) = \text{Sec } x$
(iii) $f(x) = \tan^{-1} x$

P.T.O.

(2)

(b) Calculate the derivatives:

(i) $f(x) = \sqrt{\frac{1+e^x}{1-e^x}}$ (ii) $f(x) = \log \sqrt{\frac{1+\sin x}{1-\sin x}}$ 9+6=15

4. (a) If the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is parallel to the line $x - y + 5 = 0$ find (a, b) .
- (b) Find the relative extrema of the function $y = f(x) = x^3 - 12x^2 + 36x + 8$
- (c) State Rolles Theorem and give a geometric explanation.
- (d) Verify Rolles Theorem for the function $f(x) = x^2 - 5x + 6$ on the close interval $(2, 3)$. 3+4+4+4=15

Unit-II

1. (a) Define inner product and length of a vector. How are the concepts connected?
- (b) For any two vectors X and Y is \mathbb{R}^n show that $||x|| - ||y|| \leq ||x - y||$
- (c) Write the vector $v = (1, -2, 5)$ as a linear combination of the vectors $u_1 = (1, 1, 1), u_2 = (1, 2, 3), u_3 = (2, -1, 1)$.
- (d) Normalize the vector $u = \left(\frac{1}{2}, \frac{1}{5}, -\frac{1}{3}\right)$.
- (e) For vectors $u = (1, 1, 0)$ and $v = (1, 2, 1)$ determine whether the angle between them is acute or obtuse or right and then calculate the angle. 3+4+4+2+2=15
2. (a) What is parametric representation of a line? Write the parametric equation of the line through $P_1(3, 0)$ and $P_2(5, 0)$
- (b) Write the non-parametric equation of the line $x = 3 - 4t, y = 1 + 2t$.
- (c) Find the point-normal equation of the plane P which contains the points $p = (2, 1, 1), q = (1, 0, -3)$ and $r = (0, 1, 7)$.
- (d) Find a non-parametric equation of the plane through the point $(1, 2, 3)$ and normal to the vector $(1, -1, 0)$. 4+3+4+4=15
3. (a) State the conditions for a system of two non-degenerate linear equations in two unknown to have
- (i) One Solution (ii) No Solution (iii) Infinite number of solutions with diagrams.
- (b) Solve the following system of linear equations using forward elimination and backward substitution.
- $x + 2y - 3z = 1, 2x + 5y - 8z = 4, 3x + 8y - 13z = 7.$

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(c) Consider the system.

$$x + 2y + z = 3$$

$$ay + 5z = 10$$

$$2x + 7y + az = b$$

(i) Find those values of 'a' for which the system has a unique solution.

(ii) Find those pair of values (a, b) for which the system has more than one solution.

$$6+4+5=15$$

4. (a) Let $A = \begin{bmatrix} 2 & -2 & 2 & 1 \\ -3 & 6 & 0 & -1 \\ 1 & -7 & 10 & 2 \end{bmatrix}$, Reduce A to echelon form using the pivoting algorithm.

(b) Reduce the following matrix to row Canonical form.

$$A = \begin{bmatrix} 2 & 2 & -1 & 6 & 4 \\ 4 & 4 & 1 & 10 & 13 \\ 8 & 8 & -1 & 26 & 23 \end{bmatrix}$$

(c) Solve the following system using its augmented Matrix M, $x + 2y - z = 3$,
 $x + 3y + z = 5$, $3x + 8y + 4z = 17$ 5+5+5=15
